

# A Proposed Precalculus Text<sup>1</sup>

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We are writing a precalculus text under the auspices of a National Science Foundation grant, and are interested in discussing it with a commercial publisher. At the Community College of Philadelphia (CCP), where we are using the materials, precalculus is a two- semester sequence covering most topics included in a standard precalculus book, from review of algebra and concept of function through trigonometry.

We have been working for about six years on materials which Prof. Darken and several other faculty have used in special sections at CCP. Prof. Ligare team-taught one such section with Prof. Darken. Feedback from other faculty has overall been quite good.

We started to develop these materials under the auspices of a grant from the William Penn Foundation (Minority Scholars Program), and continued with support from the National Science Foundation (as part of the Middle Atlantic Consortium for Mathematics and its Applications across the Curriculum).

In both grant projects there has been an emphasis on integrating mathematics with its applications, and we continue to develop applications and design experiences for the students to show them connections between mathematics and other aspects of education and life.

We also strongly promote cooperative learning, and we use computers as well as calculators and graphing calculators. (The software we use is Maple.)

We have worked with small classes (20 maximum) that meet four hours a week instead of the usual 3, in a computer classroom, and employ a student assistant. These unusually auspicious circumstances have enabled us to monitor the students and pay close attention to their performance and feedback. This academic year Darken and another instructor have used the materials in a modified way with standard classes of 36 students. This seems to be going well.

The students in the first course of the precalculus sequence typically arrive weak in algebra and passive in attitude. Long before the end of the semester they become much more engaged and active, and have improved their algebra skills through a "just in time" approach. We include what we call "algebra pushups"-really review-in their homework.

We believe that the exercises we have developed work well for these students. They provide a challenge at the right level. We have conducted attitude surveys at the beginning and end of each semester, which give positive results, and we are in the process of gathering follow-up data.

We have de-emphasized certain topics and rearranged the order of others. For example, we do not start, as many authors do, with a discussion of the types of numbers (natural, integer, rational, etc.) because it is our experience that the students do not believe the terminology is of any importance, and (consequently, we think) have trouble learning and retaining it.

Other innovations include:

1. A rearrangement of topics in trigonometry. We start with the geometry of triangles (recalling theorems about congruence and similarity) and do solutions of triangles before we work on periodicity of trig functions or their definition in terms of the unit circle. Trigonometry thus begins with topics that are more concrete and concepts that are more basic, and begins the subject with explicit reference to material the students have seen before.
2. Trigonometry before exponents and logs. The main reason for this is that the hardest part of the course for most students is logarithms, and a significant part of the difficulty seems to be due to a lack of ease with inverse functions generally. With this ordering, students work with inverse trig functions before logs, gaining more experience and, we hope, ease. Also, inverse trig functions arise in a pragmatic way in solving triangles. Our students' first experience with inverse trig functions is in this context, using a calculator, and they find it no big deal.

Prof. Darken has perused new precalculus texts to see if one would be suitable for the work that we do, either to replace or supplement our materials (which have not had much expository material). For one reason or another, none seemed suitable.

The traditional texts don't work very well for us because the order of topics is different from ours, and the style and language are sufficiently different to make some difficulty for the students. Also, we feel that we have developed insight into some of the ways people learn this subject, and that these texts and these ways don't match well.

A number of reform texts provide ideas and examples for use in class. One, by David Wells and Lynn Tilson, was of particular interest, but probably the reading level would be uncomfortably high for our population. The Harvard

text has some very interesting problems, but is not set up for the modified discovery approach that we prefer. Shelly Gordon's book differs from the standard approach in ways that look promising but to an extent that might well require a new course sequence, and we prefer not to do that.

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