MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Chapter 1— Graphing and Lines Instructor's Notes

1.1 Key concepts

- Collection and organization of quantitative data.
- Plotting of data.
- Relating mathematical rates of change to real-world phenomena
- Linear and piecewise linear graphs

1.2 Summary

We start with an activity for students involving stocks. We take an hour and a half or so for this when time permits, but it can be compressed. Students are given about two weeks' worth of the business section from recent newspapers, and the name of a stock to track. Their job is to extract from the newspapers the closing price of the stock over the given time period, to tabulate and graph it, and to answer some questions that require reading the graph accurately and interpreting it, especially with respect to increase and decrease, maxima and minima.

We continue with work involving motion sensors. Students move in front of a sensor which produces a distance $v.$ time graph on the screen of a computer or graphing calculator. They try to make a graph that matches a template on a transparency placed in front of the screen.

The activities and exposition are interspersed, with most class time going to activities.

1.3 Class Outline

- We begin every class with a quiz, and assign review algebra for homework most days
- Very brief description of stock market and how to read the quotes in the newspaper

- Students work in groups to gather data from papers, graph data and do exercises based on the data
- Summarize findings
- Brief lecture on slope formula, the main example being velocity as distance over time as main example
- Student work in groups duplicating templates with motion sensor, writing up results
- Brief lecture on lines (slope, equation)
- Students work on exercises pertaining to lines
- Brief lecture and class work on horizontal and vertical lines, parallel and perpendicular lines

This all takes us about 10 hours, which we are working on compressing.

1.4 Purposes

The stock exercise, which we do on the first day of class in the first-semester precalculus, serves several purposes:
Mathematical: The stock activity provides a review of basic plotting and graphing, as well as a little elementary work with signed numbers, using all these as tools to a goal–study of the stock's behavior. Students start analyzing graphical information, in particular with respect to maxima, minima and rates of change. They also get a little easy review of signed numbers.
Organizational: The stock activity starts the students with an application, thereby setting a tone, we hope. Most of our students are not familiar with stocks; nevertheless, they have heard the term, and perhaps seen TV footage of a stock exchange on some dramatic occasion. The fact that the information comes out of a current local newspaper lends a degree of immediacy and reality to the work. The activity requires some social and physical activity on the part of the students, and encourages a certain amount of initiative and group organization. It seems the right mixture of direction and independent activity for these students on the first day. (Few of them have much familiarity with independent activity in mathematics classes.) It serves as an informal diagnostic tool to assess the ability of the class as a whole, and

provides scope for observing student habits and attitudes. If there are students with organizational tendencies, these may come out in the effort to find every newspaper, and this is a good characteristic to spot early when doing group work. Most students enjoy the activity.

The motion sensor work gives students an understanding in an immediate way of what a graph does. It also requires interaction among students, continuing the effort to convey that mathematics is a real and human activity.

The exposition on lines is traditional. We tie it to the activities with the motion sensor. We have been doing the motion sensor exercises before the work on lines, but are currently trying the reverse, in order to mix the more routine material more evenly with the activities.

1.5 Materials required

For stock exercise:

1. Two weeks worth of the business section of a newspaper, for stock quotations. (It's okay if one day is missing.)

2. Graph paper (not essential)

3. If you don't want to take the class time for all students to gather all data, make a chart with the dates to be used for tracking the stock, and a column for closing price, to be filled in by students. (Some of the flavor remains if you assign groups and give each group a newspaper, from which they read the chosen stock's closing price for the day before, and report it to the rest of the class, so everybody has a complete record. You could of course just give a filled-in list, but this would take away some reality and immediacy.)

For motion sensor exercise:

1. Computers or graphing calculators with motion sensors attached

2. Overhead transparencies with simple graphs on them to be placed over the screen and used as templates for students to match by appropriate movement.

3. Instruction sheet for motion sensor activity

Note: if you don't have motion sensors, you could describe the idea, give students graphs, and ask them to describe the motion that would produce such a graph. It would seem desirable that some students, at least, actually execute the motions. Perhaps each group or the class as a whole could direct a student as to how to move.

1.6 Advance preparation for the activities

1.6.1 Stock:

You need to start two weeks in advance to collect the newspapers, and shortly before the day of the activity, select the stock. One advantage of this module as a first-day activity is that it demonstrates to the students that you have been thinking about the class regularly for at least two weeks in advance, and are not just re-hashing last semester's notes. Hopefully they are impressed by your dedication and caring.

What to look for in a stock: To make the mathematical points, you need a stock that has several ups and downs in the time interval under consideration and that does not have its two days of greatest increase in price culminate in its maximum price, nor its two days of greatest decrease culminate in its minimum price. (The exercises ask students to decide what would be the best day to purchase stock if it is to be held for exactly two days then sold. Many start by thinking the best day is always two days before it hits its peak, a notion best not reinforced.)

There are web sites that give tables and graphs of the closing price of stocks. One we've used is *www.bigcharts.com*. (Before we found the websites we looked in the newspapers for stocks with appropriate track records. This was often time-consuming.) Be aware that the abbreviations used in the newspaper listings are not always the same as the symbols needed to get the information on websites. The *Wall Street Journal* gives the symbols you need (under "Sym"). If you are missing an issue (other than the last one), this is good, because the students learn they can figure out the price for that day from the price the day after and the change.

1.6.2 Motion sensor:

Motion sensors can be purchased for either a computer (e.g., Vernier, Distance Ranger) or a graphing calculator (e.g., TI), for less than a hundred dollars. Until recently we used computers, because we had them, and the

pictures are prettier. Graphing calculators with attachments are easier to carry around and set up, and we are now trying them in a traditional classroom setting.

The sensor measures the distance to the nearest object directly in front of it, and produces a distance versus time graph on the monitor. Some computer software makes it possible to show velocity v. time and acceleration *v.* time along with distance v. time. There seems no point in presenting all this to students at this stage, but they might be worth coming back to (although we haven't done so).

We make simple templates on transparencies which the students tape to the computer screen (or just set in front of it). The templates should be numbered and the axes should be marked on these to match the coordinate system shown on the screen, and they should have graphs with increasing, decreasing and constant line segments on them. (Not every graph needs to have all of these). We use a time interval zero to ten, and distance zero to three or four meters. It's good to have one or two more templates than there will be groups of students.

Students need to walk backwards away from the sensor to keep an eye on the screen, and they need the space to do this, so you may need to move some furniture, or find a suitable place elsewhere if this is impossible.

1.7 What happens in our classroom

Starting the course with the stock exercise, we indicate the pile of newspapers we've brought in and say we've been collecting them for two weeks (or a group of us have, as is sometimes the case), and explain their job to them. We give the spiel about stocks that is in the materials for students.

Note that quotations involve some signed numbers; a little discussion of this is soothing, perhaps, but they can handle the task without it.

We show the students how to read the stock quotations (see the material for students), explain the general nature of the stock market, point out that the names of the stocks are abbreviated, give the abbreviation of the one they're to track, and go over the column headings with them. We assign them to groups of three or four (by proximity). We tell them they need to make a table and a graph, and that they can use their own judgment on the best way to do this –what scale, whether to show Saturday and Sunday, etc. Sometimes we give them graph paper. Later in the course it's better to leave graphing for homework whenever feasible, since it takes many students

a long time to do it. But it's good to do it in class the first time, since some students learn a lot (that one wishes they already knew) doing this in groups.

They need to realize that the closing price is for the last trading day *before* the date on the newspaper, and that there's no trading on weekends.

We remind them how to convert fractions to decimals using their calculators to divide.

Then we put some papers on each table and tell them to start.

When they have finished their tables, we check the figures, and they graph. Then they work on the exercises provided. We go over some of these.

For the motion sensor activity, the students divide into groups of about four (which we assign more or less randomly.) We do a quick demonstration of the way the sensor works and give them templates and instructions. Then students take turns, so each one has at least one chance to walk in front of the sensor. Of the other students, one should be assigned to start the data collection by clicking the mouse, one to describe in writing how the moving student moves to match the template, and one to see that the student doing the walking doesn't accidentally back into someone. Each group should turn in its written description of the motion required for each template. This is one reason for numbering the templates. The other is to help the groups keep track, since they need to trade the templates around so that each group does all. They should describe the motion in terms of meters per second toward or away from the sensor.

We go over the slope formula in connection with velocity, referring to one of the motion sensor templates.

1.8 Discussion

These activities explicitly address some of the mathematical ideas of rates of change in contexts that are of interest to the students (money, the motion of one's body). The collection of data from the newspapers also serves as a good mixer and introduces the collaborative style of the course that we encourage. So does the motion sensor work. For our students the degree of choice (re units, skipping weekends, methods of labeling) seems to provide the right scope for individual decision-making at this point. The exercises also serve as a good diagnostic. As we walk around the room talking to students and answering questions it's easy to see who is having problems with basic skills such as graphing and calculating percentages.

1.8.1 Stocks

For the stock exercise we raise questions: Does it make sense to connect the data points with lines? What range of values should be plotted on the vertical axes, and what effect does the choice have on the impression that a graph makes on the viewer? What should be done about weekends? The business section usually has other graphs with a variety of styles that may be worth displaying and discussing.

It takes our students about an hour and a half to collect the data and make a good graph. There are several shortcuts that some of them discover. Collaboration always helps, of course. Each day's stock listings include the net change of the price during the day, so that the students can actually get 2 days worth of data from a single paper. The determination of the previous day's price at closing using this information is a good exercise for some the students.

The day after the students work on the stock exercise we display several of the graphs created by the students and discuss the variety of ways that the data can be displayed. We have sometimes constructed a number of different graphs of the stock's performance using *Excel.* Just graphing the information using various different scales is helpful to students who are not facile at reading graphs. Also, bar graphs, pie graphs, etc., illustrating the same information may help them get a sense of variety and choice, and kick out of the "one right answer" mode.

The stock we used in January, 1998 was America On Line (chosen by colleague Geoff Schulz), with readings as follows.

Price of AOL Stock on 10 days in January 1998										
Day	1	2	3	4	5	6	7	8	9	10
Price	\$90	$\$88\frac{1}{16}$	$\$86\frac{3}{4}$	$\$86\frac{15}{16}$	$\$86\frac{5}{8}$	$\$88\frac{5}{16}$	$\$90\frac{1}{2}$	$\$89\frac{5}{8}$	$\$91\frac{1}{8}$	\$90

The motion sensor exercises take up where the stock left off in terms of analyzing graphs. These exercises seem to give students a stronger understanding of what a graph does. Some students at first expect the line to stop when they stop. Now and again a student when asked later on a test to describe the motion necessary to produce a graph will say that an interval of increase is produced by going uphill. We hope these rare cases illustrate how much the rest have learned, and what errant notions have been dispelled.

Many students enter our classes in passive mode, without much expectation of any working connection with other students, and the active work with motion sensors helps change their attitude, we think. However, while some classes take to group work easily, others are slow and hesitant. This second "get-up-and-move-around" exercise expedites matters, we think, but some classes will still need encouragement to work cooperatively. Doing so seems very efficient pedagogically, since it strongly promotes paying attention, verbalizing concerns, giving explanations, and forming human links related to mathematics. These in turn help with making sense of it.

1.8.2 Coordinate Systems and Plotting Points

This material is largely review, and should be unnecessary for well-prepared students. However, we have never had the impression we were boring the class by going over it.

Some students arrive in this course not sure how to plot a point. But they all seem to pick it up in the course of doing the stock exercise, and often without any need to reveal their initial ignorance. The group work probably helps with this–they can watch over someone's shoulder, and it may make more sense in this context than when they saw a teacher do it (as they surely did).

Also, some students don't see any need for straightness or uniformity of scale when drawing coordinate systems. There is a common tendency to make the distance from 0 to ± 1 on both axes bigger than the distances between other pairs of consecutive integers, probably in an effort to relieve congestion around the origin. It's a good thing to have students draw coordinate systems to see how they do it: if they think it's okay that the units aren't evenly spaced, they need to be disabused of this notion. But it often take them a very long time to draw a coordinate system, more time than you would want to devote to the activity very often in class, especially on a test. So after the initial practice with drawing coordinate systems and choosing scales, we usually give them the coordinate systems.

Piaget had something to say about topological notions preceding quantitative ones, and when you think about it, yes, the topology is okay here, and so is the ordering. We don't start with nothing.

1.8.3 Lines

Again, this is a section that addresses issues a well-prepared student would find easy.

Finding the slope of a line and the equation of a line involve simple routines that usually don't give much trouble. Students are used to dealing with simple routines; they think that's what mathematics is, or is supposed to be. But they do not necessarily have a grasp of the constancy of slope. We discuss this in the next section. And they do not necessarily think of the graph as corresponding to the equation in the same way we think of it. After all, there are a variety of different-looking graphs and a variety of different-looking equations all representing the same point set, and the concept of equivalence may not have set.

The topic of lines is a good one for getting students to start thinking about what's immutable in a graph and what's not; e.g., straightness of lines and the issue of whether they're rising or falling, versus scale.

There are some troubles here. It has happened on the second day that a student draws a graph by hand and the scale is so irregular that what ought to be straight lines show significant bends. This doesn't take long to clear up.

The rising and falling question is harder. If you draw on the board a graph with lots of bends in it, students enjoy telling you as you move left to right with the chalk where the graph changes from increasing to decreasing or vice versa. However, they do not as a group see to keep a firm hold of the concept. It seems to get mixed up with positive and negative values of the dependent variable.