

MATH 161 — Precalculus<sup>1</sup>  
Community College of Philadelphia

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## Math 161 — Chapter 2

### Class Exercises

1. If  $f(x) = 3x - 1$  and  $g(x) = 2x + 1$ , find
  - (a)  $f(4)$
  - (b)  $g(8)$
  - (c)  $f(2) + g(2)$
  - (d)  $f(7) - 3g(7)$
  - (e)  $(f(5) + g(5))^2$
  - (f)  $f^2(5) + 2f(5)g(5) + g^2(5)$
  - (g) Which is larger,  $f(5)$  or  $g(5)$ ?
  - (h) Which is larger,  $f(1)$  or  $g(1)$ ?
  - (i) Which function has the larger slope?
  - (j) Which function has the larger  $y$ -intercept?
2. Given  $f(x) = 2x + 3$ ,  $g(x) = 7 - x$ , and  $h(x) = x^2$ , find:
  - (a)  $f(7)$
  - (b)  $g(-1)$
  - (c)  $h(5)$
  - (d)  $f(6) + g(6)$
  - (e)  $f(2) - g(2)$
  - (f)  $h(2)$
  - (g)  $g(\pi) + h(\pi)$
  - (h)  $f(7)/g(7)$
  - (i)  $f(1)g(1)$
  - (j)  $(f(5) - h(5))^2$
  - (k)  $f^2(5) - 2f(5)h(5) + h^2(5)$

3. Let  $f(x) = 2x + 9$ . Give:

- (a) the  $y$  coordinate of the point on the graph with  $x$ -coordinate 3
- (b) the  $y$  coordinate of the point on the graph with  $x$ -coordinate -1
- (c) the  $y$  coordinate of the point on the graph with  $x$ -coordinate 6
- (d) the  $y$  coordinate of the point on the graph with  $x$ -coordinate 101
- (e) the  $x$  coordinate of the point on the graph with  $y$ -coordinate 9
- (f) the  $x$  coordinate of the point on the graph with  $y$ -coordinate 11

4. Suppose the function  $f$  is given by the table:

$x$	-2	-1	0	1	2	7
$y$	-4	3	2	4	5	0

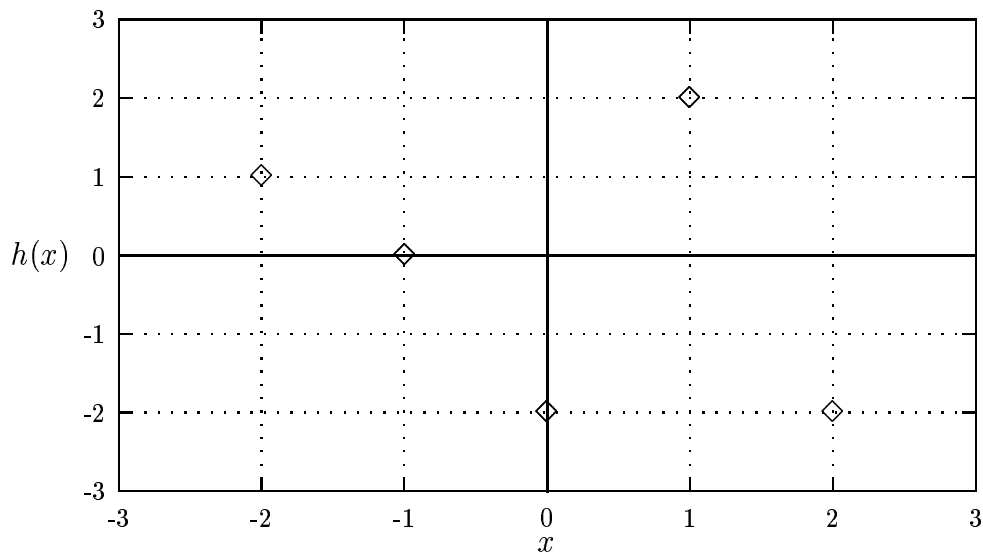
- (a) Give the domain of  $f$ .
- (b) Give the range of  $f$ .
- (c) Give:
  - i.  $f(-1)$
  - ii.  $f(0)$
  - iii.  $f(1)$
  - iv.  $f(2)$
  - v.  $f(7)$

(d) Make a graph of  $f$ .

5. Let the function  $g$  be defined by  $g = \{(1, -1), (0, 3), (6, 1)\}$ .

- (a) Give the domain of  $g$ .
- (b) Give the range of  $g$ .
- (c) Give:
  - i.  $g(1)$
  - ii.  $g(6)$
  - iii.  $g(0)$

6. Suppose the graph below defines the function  $h$ .

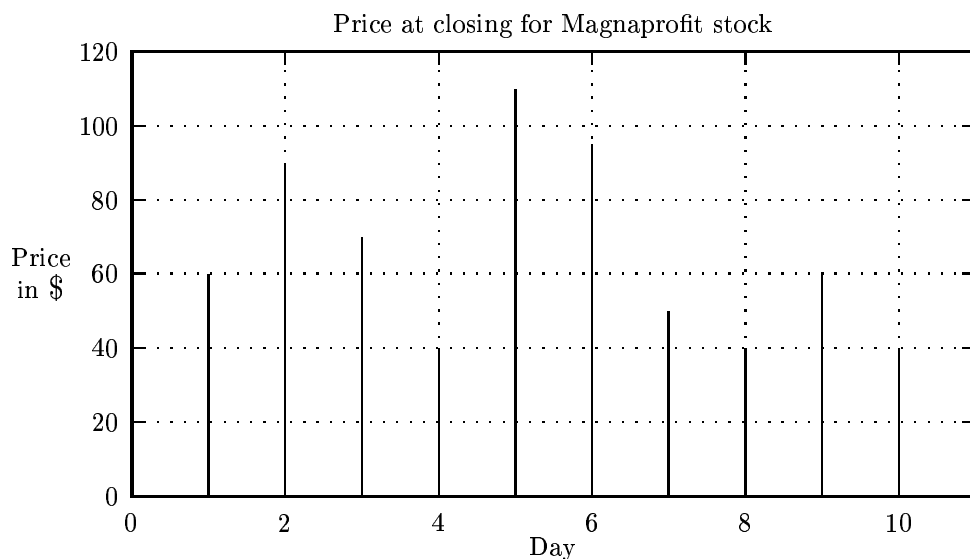


- (a) Give the domain of  $h$ .
- (b) Give the range of  $h$ .
- (c) Give:
  - i.  $h(-2)$
  - ii.  $h(2)$
  - iii.  $h(0)$
  - iv.  $h(1)$
  - v.  $h(-1)$
- (d) Write  $h$  as a set of ordered pairs.
- (e) Write  $h$  in the form of a table.

For the next three exercises you will be combining your results on this exercise with the results of others, as your instructor directs. (Every individual or group needs to use the same coordinate system, on the board or a sheet of paper, or else each group uses a transparency, all the transparencies with the same coordinate system drawn on them.)

7.
  - (a) List and plot 15 points – any points you want that can be shown on the coordinate system given.
  - (b) Mark the points with  $x$ -coordinate greater than 3 in blue.
  - (c) Mark the points with  $x$ -coordinate greater than 3 in red.
  - (d) Compare the results of all individuals or groups.
  - (e) Write an inequality describing the set of blue points
  - (f) Write an inequality describing the set of red points
8.
  - (a) List and plot 15 points with  $y$ -coordinate greater than 2.
  - (b) Compare results of all individuals or groups.
  - (c) Write an inequality describing the points obtained.
9.
  - (a) List and plot 15 points with  $x$ -coordinate less than -1.
  - (b) Compare results of all individuals or groups.
  - (c) Write an inequality describing the points obtained.
10. Sketch the set of all points  $(x, y)$  such that  $x > -4$ .
11. Sketch the set of all points  $(x, y)$  such that  $y < 5$ .
12. Sketch the set of all points  $(x, y)$  such that  $x \geq -4$  and  $y \leq 5$ .
13. Sketch the set of all points  $(x, y)$  such that  $x + y < 6$ .

14. Below is a graph of the closing prices of Magnaprofits stock over a period of ten trading days. Answer the following questions referring to the graph.

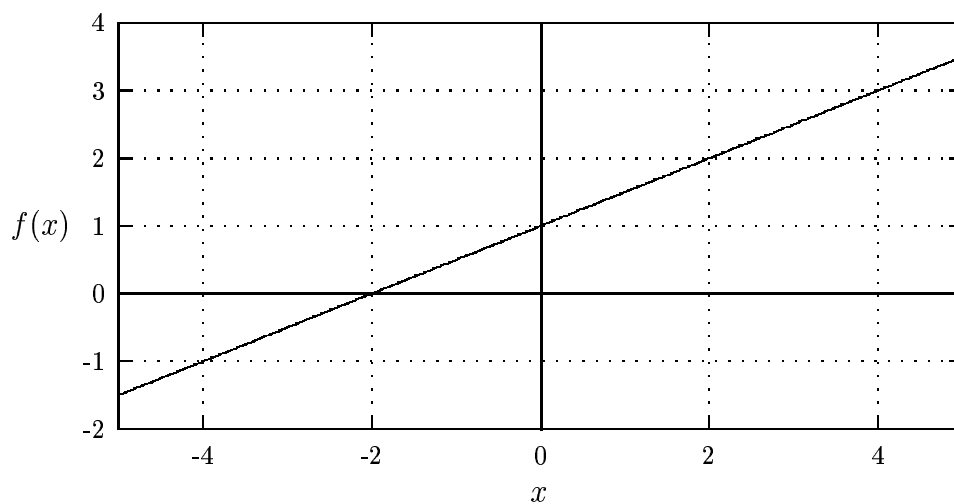


- (a) Suppose Mr. Less gives his broker orders to buy 100 shares of Magnaprofits on any day the price is below \$50 per share. Does he buy any stock? If so, on what days? How much does he pay for all the stock he buys?
- (b) Suppose Ms. More gives her broker orders to sell 10 shares of her Magnaprofits stock on any day the price is \$80 a share or more. Does she sell any stock? If so, when? How much does she get for all the stock she sells?
15. Consider the points  $P = (1, 5)$ ,  $Q = (2, 9)$ ,  $R = (-4, 2)$ ,  $S = (0, 8)$ .
- (a) Plot the points.
- (b) List these points in order from the one with the largest  $y$ -coordinate to the one with the smallest  $y$ -coordinate.
- (c) List these points in order from the one with the largest  $x$ -coordinate to the one with the smallest  $x$ -coordinate.

- (d) List and plot 3 points (any 3 points you choose) with  $x$ -coordinate greater than that of  $P$ .
  - (e) Sketch the set of all points in the plane that have  $x$ -coordinate greater than that of  $P$ . e) Write an inequality that describes the set of all points in the plane that have  $x$ -coordinate greater than that of  $P$ .
  - (f) List and plot 3 points (any 3 points you choose) with  $x$ -coordinate greater than that of  $R$  and less than that of  $Q$ .
  - (g) Sketch the set of all points in the plane that have  $x$ -coordinate greater than that of  $R$  and less than that of  $Q$ .
  - (h) Write an inequality that describes the set of all points in the plane that have  $x$ -coordinate greater than that of  $R$  and less than that of  $Q$ .
  - (i) List and plot 3 points (any 3 you choose) with  $y$ -coordinate greater than that of  $R$  and less than that of  $Q$ .
  - (j) Sketch the set of all points in the plane that have  $y$ -coordinate greater than that of  $R$  and less than that of  $Q$ .
  - (k) Sketch the set of all points in the plane that have  $y$ -coordinate greater than that of  $R$  and less than that of  $Q$ .
16. Write the formula of a piecewise function satisfying the given conditions. The domain of each function should be all real numbers.
- (a) The graph consists of two horizontal pieces, and the points  $(0, 4)$  and  $(3, -2)$  lie on the graph.
  - (b) The graph consists of two linear pieces meeting at  $(3, 5)$ .
  - (c) The graph consists of two linear pieces, one with positive slope and one with negative slope, and the pieces meet at  $(0, 0)$ .
  - (d) The graph consists of two linear pieces, is not continuous (that is, the pieces don't meet), and the function has negative outputs for negative inputs, positive outputs for positive inputs.
  - (e) The graph consists of three linear pieces, one of which is horizontal, one of which has positive slope and one of which has negative slope, and the graph is continuous (that is, no breaks and the pieces do meet).

17. The graph of a function  $f(x)$  is illustrated below. Use the graph to complete the following exercises.

Problem 17

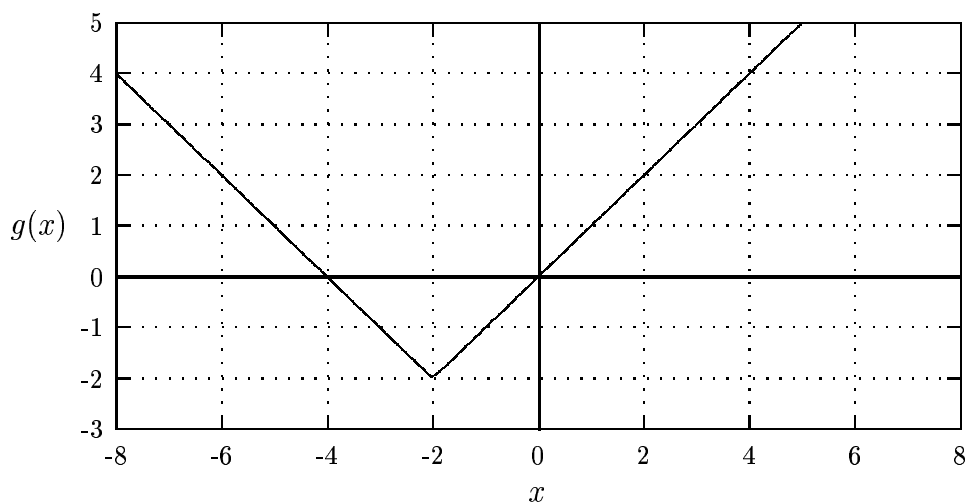


- (a) Estimate  $f(4)$
- (b) Estimate  $f(-4)$
- (c) Estimate  $f(-1)$
- (d) Give an interval over which the function  $f(x)$  is always greater than zero.
- (e) Give three values of  $x$  for which the function  $f(x)$  is less than zero.
- (f) Give three values of  $x$  for which the function  $f(x)$  is greater than 2.



18. The graph of a function  $g(x)$  is illustrated below. Use the graph to complete the following exercises.

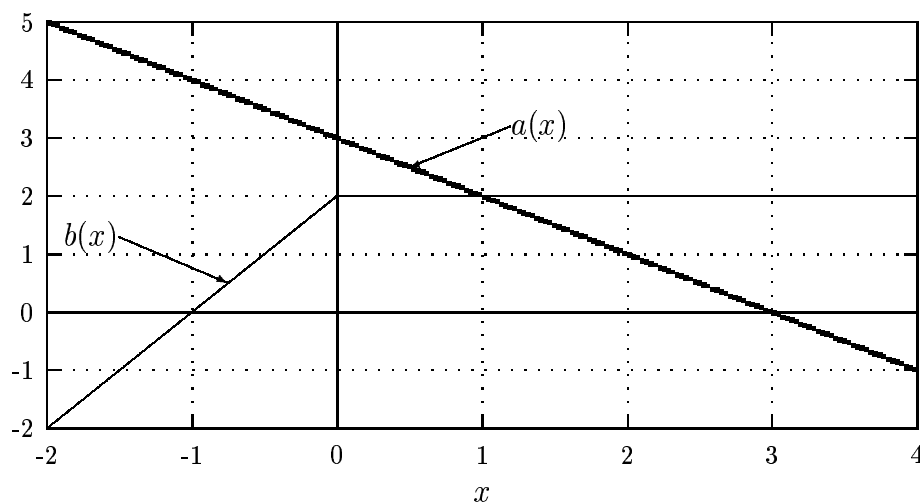
Problem 18



- (a) Estimate  $g(4)$
- (b) Estimate  $g(-4)$
- (c) Estimate  $g(-1)$
- (d) Describe, using interval notation, the complete set of  $x$  values for which the function  $g$  is greater than zero.
- (e) Describe using interval notation the complete set of  $x$  values for which the function  $g$  is greater than 2.
- (f) Describe using interval notation the complete set of  $x$  values for which the function  $g$  is less than 2.
- (g) Give an interval over which the function  $g$  is always increasing.
- (h) Give an interval over which the function  $g$  is always decreasing.

19. The graphs of two functions,  $a(x)$  and  $b(x)$ , are illustrated below. Use the graph to complete the following exercises.

Problem 19



- (a) Estimate  $a(2)$
- (b) Estimate  $b(2)$
- (c) Estimate  $a(-1)$
- (d) Estimate  $b(-1)$
- (e) Estimate  $a(1)$
- (f) Estimate  $b(1)$
- (g) Estimate  $a(0) + b(0)$
- (h) Estimate  $b(3) - a(3)$
- (i) Give an interval over which the value of the function  $a$  is always greater than the value of the function  $b$ .
- (j) Give an interval over which the product of the functions  $a$  and  $b$  is always positive.
- (k) Give an interval where the slope of the function  $a$  is less than the slope of the function  $b$ .
- (l) Sketch a graph of the function  $w(x) = b(x) + 2$ .
- (m) Sketch a graph of the function  $w(x) = a(x) - 1$ .

20. In problem 19 the graphs of the functions  $a(x)$  and  $b(x)$  are illustrated. Consider the function  $c(x) = a(x) + b(x)$ . On graph paper make a sketch of the function  $c(x)$ .
21. Sketch the graphs of the functions  $f(x)$  and  $g(x)$  from problem 1. Put both functions on the same coordinate system and make sure your  $x$  values go from -2 to 6.
- (a) By looking at your graph, determine which is larger:  $f(-1)$  or  $g(-1)$ .
- (b) By looking at your graph, determine which is larger:  $f(5)$  or  $g(5)$ .
- (c) By looking at your graph, determine which is larger:  $f(2)$  or  $g(2)$ .
22. Sketch the graph of each of the following functions:

(a) 

$x$	1	2	3	4	5
$y$	7	8	9	2	1

(b) 

$x$	1	2	3	-1	-2
$y$	-2	2	9	2	1

(c)

$$f(x) = \begin{cases} 3 & \text{if } x < 2 \\ 7 & \text{if } x \geq 2 \end{cases}$$

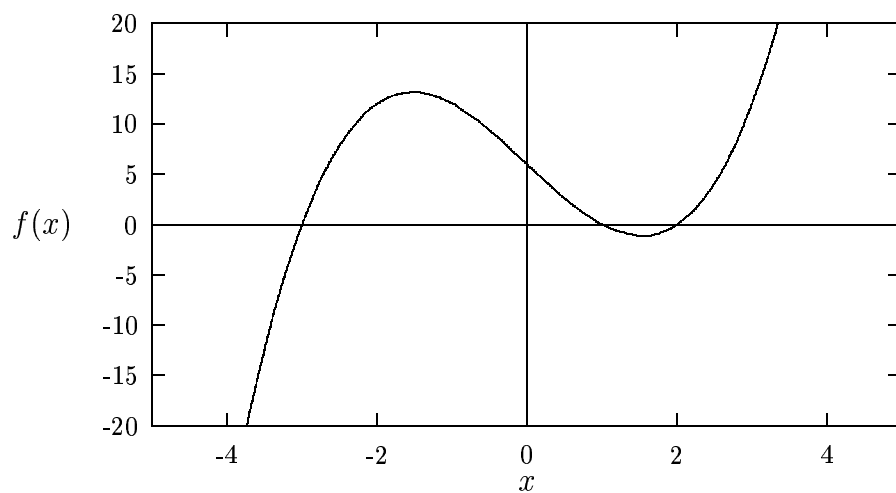
(d)

$$g(x) = \begin{cases} 2x & \text{if } x < 4 \\ 8 & \text{if } x \geq 4 \end{cases}$$

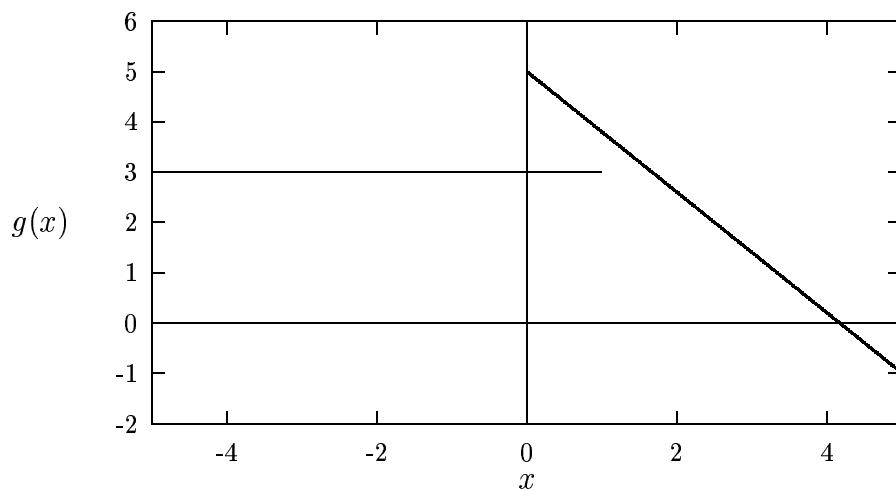
23. Juan the Walker walks along a straight highway at a speed of 3 miles/hour. He starts at time  $t = 0$  at mile marker 4 and walks toward higher numbered mile markers. His position along the highway as a function of time can be written as a function  $p(t)$ . Find the following values of the function:
- (a)  $p(0)$
- (b)  $p(1)$
- (c)  $p(3)$

For each of the correspondences given in Problems 24-33 determine whether the correspondence defines a function. If the the correspondence does not define a function, give a specific reason why it doesn't.

24.



25.



26.

$$f(x) = \begin{cases} 3 & \text{for } x < 2 \\ 7 & \text{for } x \geq 2 \end{cases}$$

27.

$$g(x) = \begin{cases} 2 & \text{for } x < 4 \\ 3 & \text{for } x > 1 \end{cases}$$

28.

$$h(x) = \begin{cases} x & \text{for } x < 0 \\ 7 & \text{for } x > 2 \end{cases}$$

29.

$$j(x) = \begin{cases} -2x + 3 & \text{for } x < 0 \\ 3 & \text{for } x \geq 1 \end{cases}$$

30.

$x$	1	2	3	4	5
$y$	7	8	9	2	1

31.

$x$	1	3	2	5	2
$y$	-2	2	9	2	1

32.

$x$	1	2	3	2	1
$y$	7	8	1	2	1

33.

$x$	1	2	3	4	5
$y$	1	8	9	2	1

34. For each of the correspondences given in problems 26-33 that **does** define a function, draw a graph of the function.

35. Consider the function  $f(x) = 4x + 3$ .

- (a) Calculate the average rate of change of the function over the following intervals:
  - i.  $[2, 3]$
  - ii.  $[-4, -2]$
  - iii.  $[-1, 2]$
- (b) What is the slope of the function  $f$ ?

36. Consider the function  $g(x) = -2x - \pi$ .

- (a) Calculate the average rate of change of the function over the following intervals:
  - i.  $[1, 5]$
  - ii.  $[-4, -2]$
  - iii.  $[-1, 3]$
- (b) What is the slope of the function  $g$ ?

37. Consider the function

$$h(x) = \begin{cases} 2.5 & \text{for } x < 1 \\ -1.5x + 4 & \text{for } x \geq 1 \end{cases}$$

- (a) Sketch the graph of the function  $h(x)$ .
- (b) Calculate the average rate of change of the function over the following intervals:
  - i.  $[1, 5]$
  - ii.  $[5, 10]$
  - iii.  $[-4, -2]$
  - iv.  $[-5, -4]$
  - v.  $[-1, 3]$

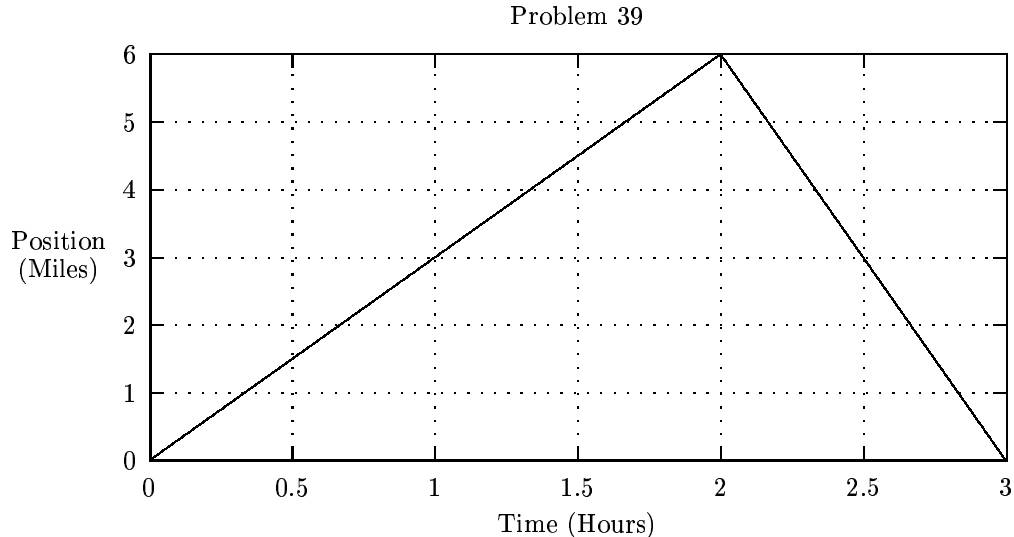
38. Find the slope of each function in 38a through 38c below, and order them from smallest to largest. Indicate if any of the functions represent parallel lines.

(a)  $f(x) = 200 - 11x$

(b)  $g(x) = 0.8x - 51.2$

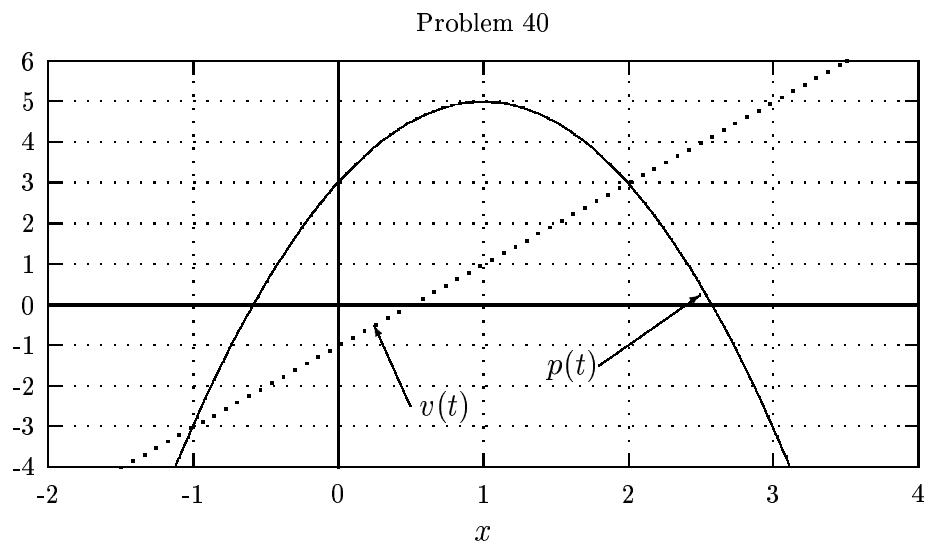
(c)  $h(x) = 15 - \sqrt{3}x$

39. The following graph illustrates a function  $p(t)$  that gives a walker's position (in miles) along a highway as a function of the time (in hours).



- (a) Calculate the walker's average rate of change of position, *i.e.*, speed, during the time interval  $[0, 1.5]$ .
- (b) Calculate the walker's average rate of change of position, *i.e.*, speed, during the time interval  $[2, 3]$ .
- (c) Calculate the walker's average rate of change of position, *i.e.*, speed, during the time interval  $[0, 2.5]$ .
- (d) Calculate the walker's average rate of change of position, *i.e.*, speed, during the time interval  $[1, 2.5]$ .

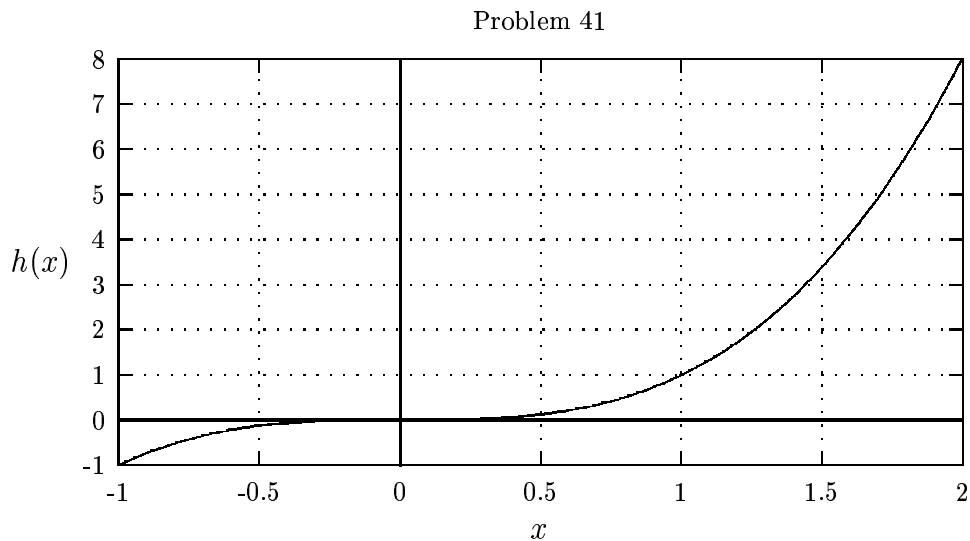
40. The following graph shows two functions  $p(t)$  and  $v(t)$ .



- (a) Calculate the *average rate of change* of the function  $v$  over the interval  $[-1, 2]$ .
- (b) Calculate the *average rate of change* of the function  $p$  over the same interval as in the previous part.
- (c) Calculate the *average rate of change* of the function  $p$  over the interval  $[0, 2]$ .
- (d) Give an interval over which the function  $p$  is increasing.
- (e) Give an interval over which the function  $p$  is decreasing.



41. The following graph illustrates a function  $h(x)$ .



- (a) Calculate the average rate of change of the function  $h$  with respect to  $x$  over the interval  $[-1, 0]$ .
- (b) Calculate the average rate of change of the function  $h$  with respect to  $x$  over the interval  $[0, 1]$ .
- (c) Calculate the average rate of change of the function  $h$  with respect to  $x$  over the interval  $[1, 2]$ .
- (d) Calculate the average rate of change of the function  $h$  with respect to  $x$  over the interval  $[0, 2]$ .

42. The following graph illustrates the same hypothetical stock data (“Stock A”) that was used in Unit 1 Homework Problem 11. Calculate the average rate of change of the stock in *dollars per day* between the close of trading on day 1 and the close of trading on day 5.

