MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Chapter 3
Class Exercises

1. Graph each of the functions below on graph paper. Show the table of values you use to get the coordinates of the specific points you plotted.

   (a) $f(x) = x^2$

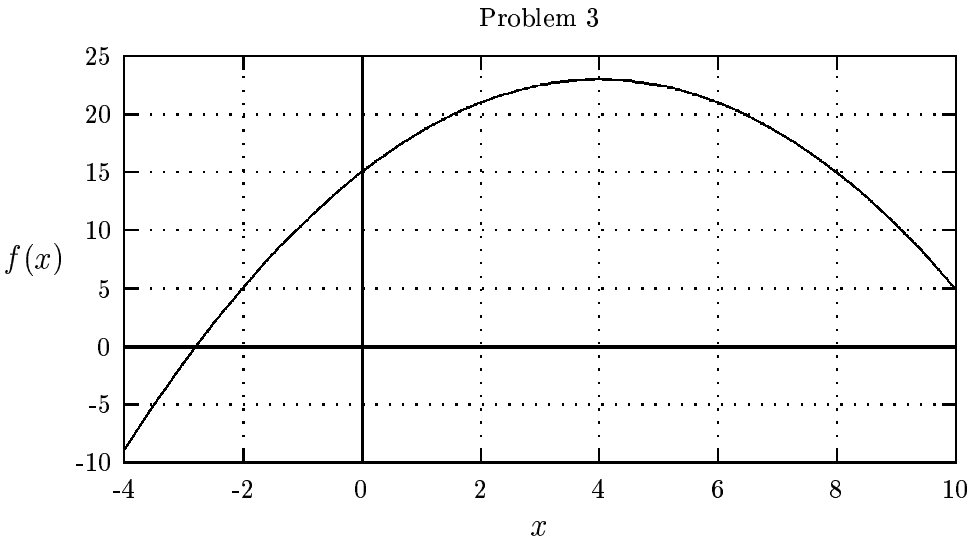
   (b) $g(x) = -x^2 + 6x$

2. Use a computer or graphing calculator to graph the same functions you plotted by hand in the previous problem. Choose several different domains and ranges to get a good feel for the overall shape of the graph.

3. The graph below is a plot of a function of the form

$$f(x) = a(x - h)^2 + k,$$

   where a, h, and k are some numbers ($a \neq 0$) that you are to determine. See how close you can come to reproducing this graph by using a computer or graphing calculator and experimenting with different values of a, h, and k. Suggestion: Start by plotting the function with the values $a = -1$, $h = 6$, and $k = 10$. Then change *one* of the numbers a, h, or k, and re-plot the function, and see the effect on the graph. Once you have figured out the effect of each of the numbers a, h, and k in this way, your job should be much easier. Write your best function and print it with your graph, if you are using a computer, or write the function with a sketch of the graph if not.



4. Graph each of the following using a computer or a graphing calculator and give the coordinates of the vertex from the graph.

   (a) $f(x) = (x-2)^2 + 7$

   (b) $g(x) = (x-3)^2 + 2$

   (c) $h(x) = 1 + (x+3)^2$

   (d) $i(x) = (x-4)^2 - 9$

   (e) $j(x) = -11 + (x+6)^2$

   (f) $k(x) = (x-30)^2 - 250$

5. In problem 4 there is a definite connection between the vertex and the formulas for the functions. Compare each formula with the corresponding vertex and identify the connection.

6. Write each of the functions in Problem 4 in the form $f(x) = ax^2+bx+c$, where a, b, and c are numbers. Give your values for a, b, and c for each of the functions.

7. Which of the following functions are quadratic functions?

   (a) $F(x) = x^2 - 91x + 130$

   (b) $G(x) = |x^2 - 7x + 6|$

   (c) $H(x) = x^2 + 0.76x + 1.32$

   (d) $I(x) = (x - 2)^2 + 4$

   (e) $J(x) = \sqrt{x^2 - 3x + 8}$

   (f) $K(x) = -13x^2 + 57$

   (g) $L(x) = 0.00003x^2 - 0.00017x + 543.22$

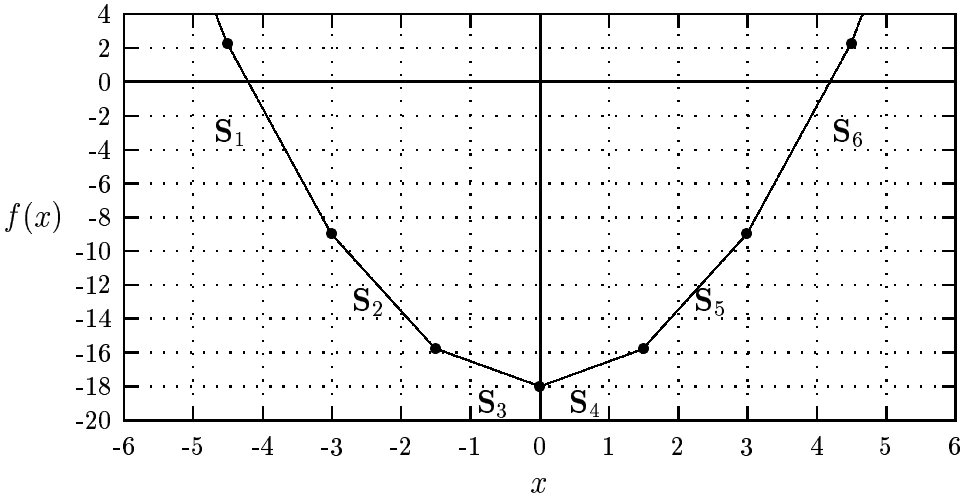
   (h) $M(x) = x^2 - 3x + 1/x$

8. Graph each of the functions in Problem 7 using a computer or graphing calculator. Which appear to have parabolic graphs? (Make sure you have selected a "window" for your graph that gives a good view of the overall appearance of the function.)

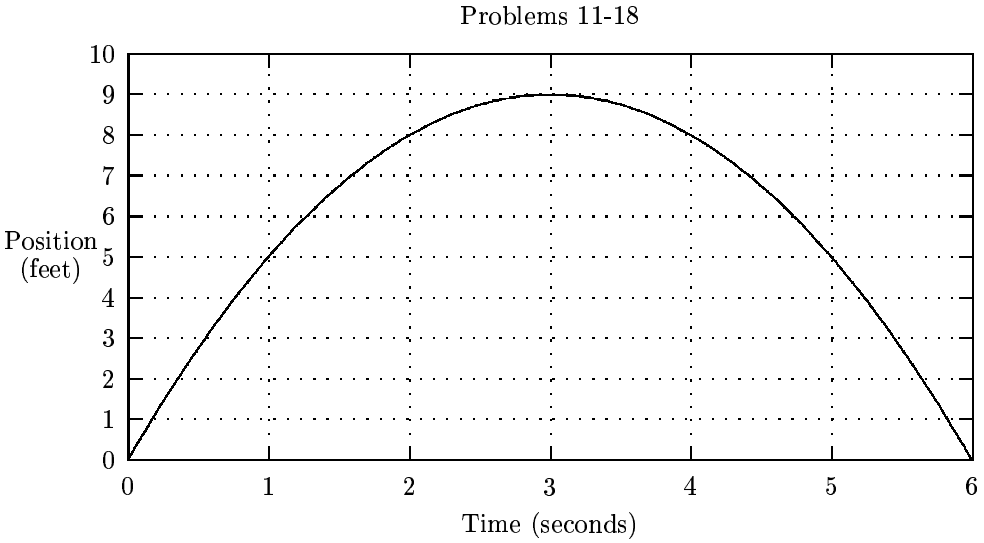
9. Based on your observation of the graphs for Problem 8, what type of formula yields a function with a parabolic graph?

10. Order the line segments $\mathbf{S_1}$ through $\mathbf{S_6}$ on the graph below by slope, from least to greatest.

Problem 10



For the questions 11-18, use the graph below illustrating the position of a ball rolling on an inclined plane as a function of time. This graph could have been produced by a motion sensor like the one you have used in class. (Positions are measured in feet and the time is measured in seconds.)



11. For what times is ball moving with positive velocity?

12. For what times is the ball moving with negative velocity?

13. When does the ball reach its farthest point along the plane?

14. How far along the plane does the ball go?

15. How long does the ball spend beyond the position $p = 5\, feet$?

16. Find a function $p(t)$ describing the position of the ball as a function of time. (This is not as straightforward as finding a function for a person walking at a constant speed. You may have to use a computer or calculator to help you.)

17. Calculate the average rate of change, or average velocity for the following intervals:

    (a) $[1, 2]$

    (b) $[2, 3]$

(c) $[3, 4]$

(d) $[2, 4]$

(e) $[5, 6]$

18. Use your results from the previous problem to make a graph in which you plot a set of points with the average rate of change over an interval as the y coordinate, and the midpoint of the interval as the x coordinate.

For the questions 19-20 consider the function $h(t) = -5t^2 + 20t + 2$ which describes the height of a ball thrown straight up into the air as a function of time. (The height is measured in meters and the time is measured in seconds.)

19. Find the average rate of change of the ball, or average velocity, for the following intervals:

    (a) $[0, 1]$

    (b) $[1, 2]$

    (c) $[2, 3]$

    (d) $[3, 4]$

    (e) $[0, 2]$

    (f) $[2, 4]$

    (g) $[1.9, 2.1]$

20. Find the average rate of change of the ball, or average velocity, in the following intervals:

    (a) $[1, 2]$

    (b) $[1, 1.5]$

    (c) $[1, 1.3]$

    (d) $[1, 1.2]$

    (e) $[1, 1.1]$

21. Find a pattern in the preceding exercise and predict what sort of aroc's you would get for intervals of the form $[1, 1 + h]$ if h is very small.

22. As a contrast to the previous problem, consider the linear function $g(t) = 10t + 2$. Find the average rate of change for the following intervals. (If you understand linear functions well this problem should take almost no time to complete.)

    (a) $[1, 2]$

    (b) $[1, 1.5]$

    (c) $[1, 1.3]$

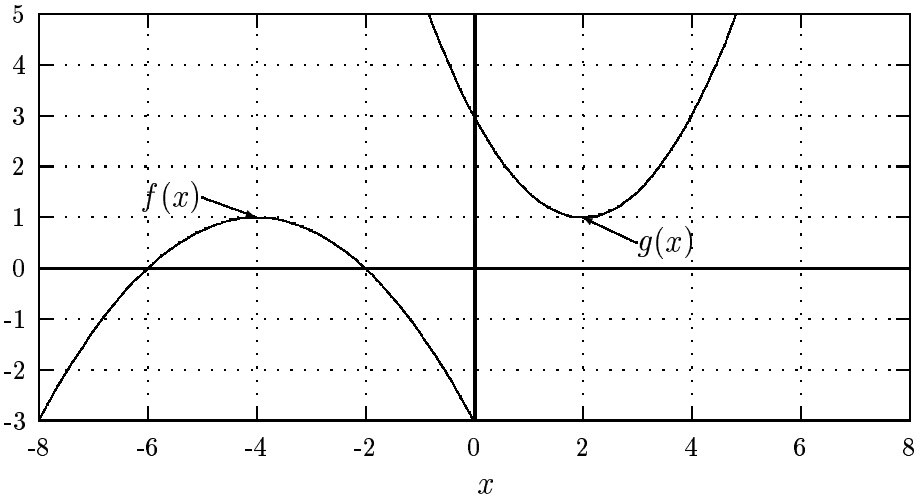
    (d) $[1, 1.2]$

    (e) $[1, 1.1]$

23. Derive a formula for the average rate of change of the function $h(t) = -5t^2 + 20t + 2$ that is good for the interval $[1, 1 + q]$, where q is an undetermined constant. Use your formula to check your answers in Problem 20.

24. Examine the graphs of the quadratic functions $f(x)$ and $g(x)$ that are illustrated below.

Problem 24



    (a) Give the y-intercepts, if any, of the function $f(x)$.

    (b) Give the vertex of the function $f(x)$.

    (c) Give all roots of the function $f(x)$.

    (d) Give the y-intercepts, if any, of the function $g(x)$.

(e) Give the vertex of the function $g(x)$.

(f) Give all roots of the function $g(x)$.

25. Find the roots (if there are any) of each of the following quadratic functions:

(a) $f(x) = (x-3)(x+4)$

(b) $g(x) = x^2 - 3x + 2$

(c) $h(x) = 5x^2 + 4x - 2$

(d) $q(x) = 3(x+1)(x-7)$

(e) $s(x) = -6x - 11 + x^2$

(f) $t(x) = x^2 - 3x + 7$

(g) $t(x) = x(x+58)$

26. For each of the following functions complete the square to find the vertex of the parabola:

(a) $f(x) = x^2 - 6x + 2$

(b) $g(x) = x^2 + 10x + 1$

(c) $h(x) = x^2 + 4x - 9$

(d) $q(x) = x^2 - 8x - 13$

(e) $s(x) = x^2 - 14x + 3$

(f) $t(x) = x^2 - 16x + 24$

(g) $f(x) = x^2 + 12x + 7$

(h) $g(x) = x^2 - 10x + 11$

(i) $h(x) = x^2 + 14x - 4$

(j) $q(x) = x^2 + x - 1$

(k) $s(x) = x^2 - 9x + 3$

(l) $t(x) = 2x^2 - 20x + 7$

27. For parts 25a to 25g in Problem 25, find the vertex and y-intercept of the parabola and graph the function.

28. For each of the following quadratic functions: *i*) identify the values of a, b, and c, *ii*) find the x-intercepts (if any), *iii*) find the y-intercept of the graph of each function, and *iv*) make a rough sketch of the function using the information you have about the intercepts.

(a) $f_1(x) = x^2 - 5x - 6$

(b) $f_2(x) = 2x^2 + 3x - 9$

(c) $f_3(x) = -2x^2 - x + 10$

(d) $f_4(x) = 3x^2 + 3x + 3$

(e) $f_5(x) = x^2 - 8x + 16$

29. Write the formula of a quadratic function satisfying the given conditions. Use your favorite letters for the names of the functions.

    (a) The graph has x-intercepts -3 and 5, and y-intercept -5.

    (b) The graph has x-intercepts 2 and 8 and y-intercept 64

    (c) The graph has x-intercepts -2 and -6 and y-intercept 24

    (d) The graph has roots 4 and -4, and y-intercept 32.

    (e) The graph has x-intercept 2 and y-intercept 8.

    (f) The graph has x-intercept -1 and y-intercept -3.

    (g) The graph has roots 0 and 4 and goes through the point $(2, -16)$.

30. Sketch the graph of a quadratic function which has a y-intercept 4 and vertex $(6, 5)$.

31. Write a formula for the quadratic function you sketched in Problem 30

32. Sketch a graph of a quadratic function for which the average rate of change over an interval is greater for intervals further to the right.

33. For each of the following descriptions of a parabola, decide whether there exists a quadratic function with such a parabola for its graph. If so, give a formula for it and sketch its graph. If not, explain why not.

    (a) vertex (3,8), y-intercept 5

    (b) roots 7 and 3 and y-intercept 21

    (c) roots -3 and 6, vertex (7,5)

    (d) roots 0 and 6 and vertex (3,9)

    (e) increasing for $x < 0$, decreasing for $x > 0$, vertex (5,2)

34. Compare the graphs of the functions given below by graphing them on the same coordinate system using a computer or graphing calculator. When you have several functions graphed on the same coordinate system, one thing you have to figure out is which graph goes with which formula. (Computer monitors may show the graphs in color, and will use the same sequence of colors each time graphs are plotted, so one way to find out is to graph some functions you know well and whose graphs you can match with the formulas.)

    (a) Test functions: $f_1(x) = 1$, $f_2(x) = 2$, $f_3(x) = 3$, $f_4(x) = 4$, $f_5(x) = 5$, $f_6(x) = 6$. After graphing these, make a list of what color is assigned to the first function, the second function, etc.

    (b) For the following sets of functions describe in words the similarities among the graphs and the differences, with special attention to the rate of change. You may want to try more than one window.

        i. $f_1(x) = x$, $f_2(x) = 2x$, $f_3(x) = 3x$, $f_4(x) = 4x$, $f_5(x) = 5x$, $f_6(x) = 6x$

        ii. $f_1(x) = x^2$, $f_2(x) = 2x^2$, $f_3(x) = 3x^2$, $f_4(x) = 4x^2$, $f_5(x) = 5x^2$, $f_6(x) = 6x^2$

        iii. $f_1(x) = -x^2$, $f_2(x) = -2x^2$, $f_3(x) = -3x^2$, $f_4(x) = -4x^2$, $f_5(x) = -5x^2$, $f_6(x) = -6x^2$

        iv. $f_1(x) = 5x^2$, $f_2(x) = 12x^2$, $f_3(x) = -3x^2$, $f_4(x) = 0.8x^2$, $f_5(x) = -7x^2$, $f_6(x) = 17x^2$

35. Use a computer or a graphing calculator to graph the functions $f_1(x) = x^2$, $f_2(x) = 10x^2$, $f_3(x) = 100x^2$. What is a good window?

36. For each quadratic function, give the requested information.

    (a) Find the roots of $f(x) = x(x - 1)$.

    (b) Find the vertex of $g(x) = 4(x + 2)^2 - 4$.

    (c) The y-intercept of $h(x) = -5x^2 + 2x - 13$.

    (d) Which way does the function h above open — up or down?

    (e) Which way does the function g above open — up or down?

    (f) Which way does the function f above open — up or down?

    (g) Find the roots of the function h above.

    (h) Find the y-intercept of the function f above.

(i) Find the roots of the function g above.

37. Give the range of the two illustrated quadratic functions, f and g. (Assume, as usual for quadratic functions, that the domain is all real numbers.)

Graph for Problem 37