MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Chapter 4 Homework

1. Make a quick sketch of each of the following power functions. Put all odd-degree power functions on the same coordinate system and label them, and do the same for the even-degree ones.

   (a) $a(x) = -x^5$

   (b) $b(x) = x^4$

   (c) $c(x) = x^6$

   (d) $d(x) = -x^4$

   (e) $e(x) = x^3$

   (f) $f(x) = x^7$

   (g) $g(x) = x^8$

   (h) $h(x) = -x^9$

2. Consider the following cubic function:

$$f(x) = 2x^3 + 4$$

   (a) Give the values of a_3, a_2, a_1, and a_0 for this function.

   (b) Graph the function by hand on a piece of graph paper from $x = -5$ to $x = 5$. Label important features on your graph such as intercepts and extremal points (maxima and minima).

3. Consider the following cubic function:

$$g(x) = -2x^3 + 2x^2 + 6x + 2$$

   (a) Give the values of a_3, a_2, a_1, and a_0 for this function.

   (b) Graph the function by hand on a piece of graph paper from $x = -2$ to $x = 3$. Label important features on your graph such as intercepts and extremal points (maxima and minima).

4. Determine which of the following are cubic functions. For those that are, give the values of the leading coefficients.

   (a) $a(x) = -3.2x^3$

   (b) $b(x) = 3 - \sqrt{5}x^3 + 4x^2 + \pi$

(c) $c(x) = (2x^3 - 5x)^2$

(d) $d(x) = x(x-3)(x+3)$

(e) $e(x) = x(x^2 - 4.2x + 3)$

(f) $f(x) = \sqrt{x^3}$

(g) $g(x) = |3 - x^2 + 2x^3|$

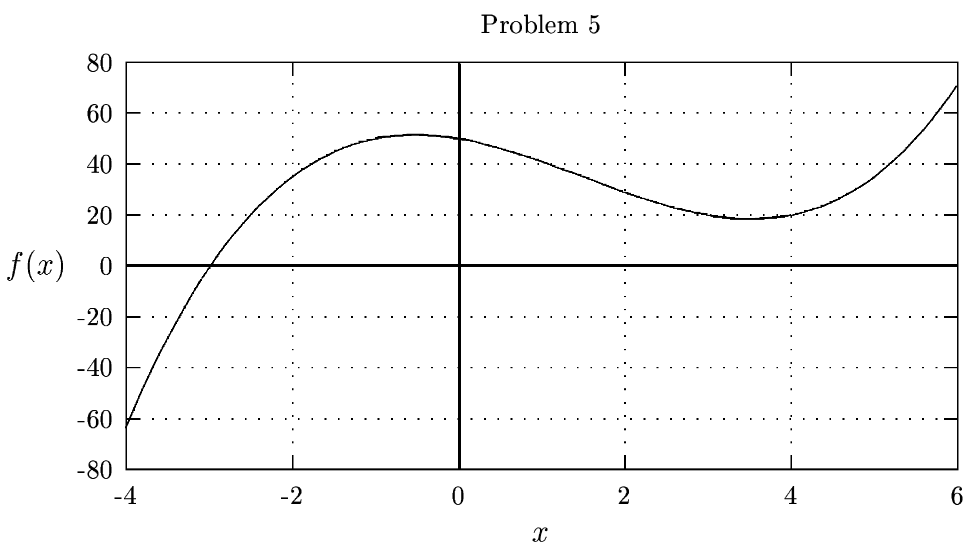
(h) $h(x) = 4x + 3.2$

(i) $i(x) = -2(3-x)^2 + x$

(j) $j(x) = (x-1)(x-2)(x-3)(x-4)$

(k) $k(x) = (x-1)^3$

5. Illustrated below is the graph of a cubic function $f(x)$.



From the graph,

(a) list approximate values of all roots,

(b) list the approximate coordinates of all local maxima and minima,

(c) give approximate values for any y-intercepts,

(d) give the limit of $f(x)$ as $x \to +\infty$; that is, when x is far to the right of zero, assuming that all trends displayed in the graph continue outside the displayed range, and

(e) give the limit of $f(x)$ as $x \to -\infty$; that is, when x is far to the left of zero, assuming that all trends displayed in the graph continue outside the displayed range.

6. Make sketches of the graphs of cubic functions that have the following properties:

   (a) Three roots, at $x = -2$, $x = -1$, $x = 1$, and a negative leading coefficient.

   (b) One root at $x = 10$, and a local maximum at $(-5, 20)$.

   (c) One root at $x = 2.5$, a y-intercept -10, and a positive leading coefficient.

7. Let $g(x) = -2.5x^3 + x - 10$. Find the average rate of change of g over each of the given intervals:

   (a) $[-4, -3]$

   (b) $[-3, -2]$

   (c) $[-2, -1]$

   (d) $[-1, 0]$

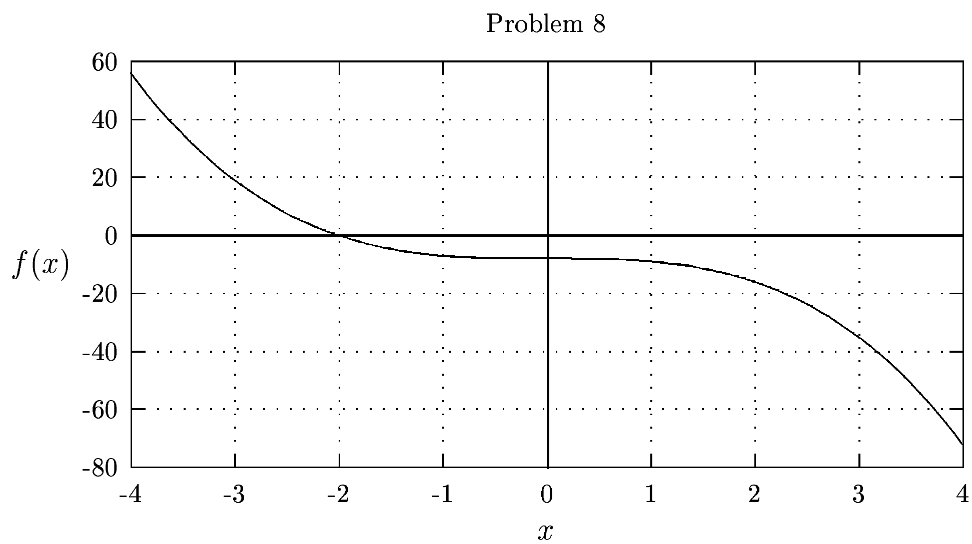
   (e) $[0, 1]$

   (f) $[1, 2]$

   (g) $[2, 3]$

   (h) $[3, 4]$

8. Illustrated below is the graph of a cubic function $g(x)$.



From the graph,

(a) list approximate values of all roots,

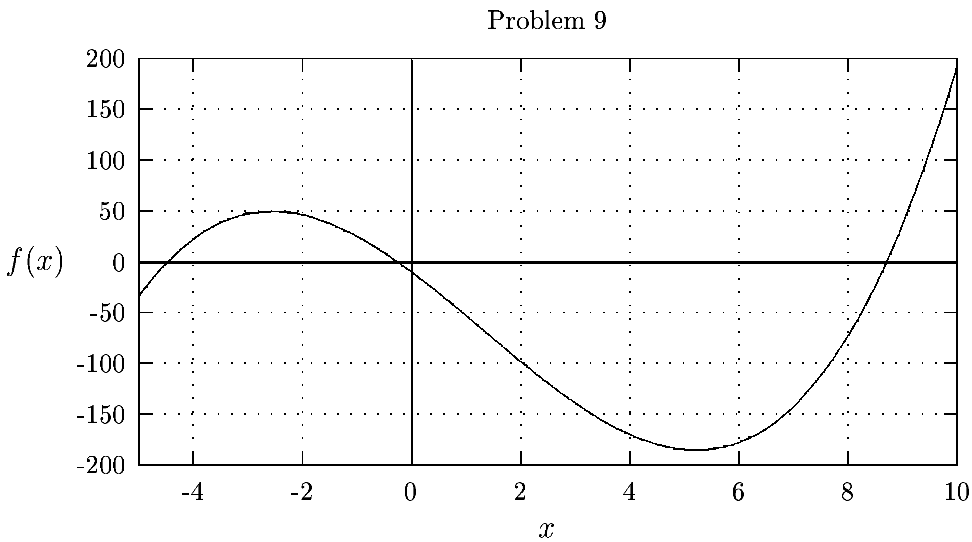
(b) list the approximate coordinates of all local maxima and minima,

(c) give approximate values for any y-intercepts,

(d) give the limit of $g(x)$ as $x \to +\infty$, that is, when x is far to the right of zero (assuming that all trends displayed in the graph continue outside the displayed range), and

(e) give the limit of $g(x)$ as $x \to -\infty$ (assuming that all trends displayed in the graph continue outside the displayed range).

9. Consider the cubic function $f(x)$ illustrated below:



From the graph, estimate the average rate of change of f over each of the given intervals

(a) $[-4,-2]$

(b) $[-2,0]$

(c) $[0,2]$

(d) $[2,4]$

(e) $[4,6]$

(f) $[6,8]$

10. For each of the following functions indicate whether or not the function is a polynomial. If it is a polynomial give the degree of the polynomial and the value of the leading coefficient.

(a) $f(x) = 8x^3 + 4x^2 + 3x - 9$

(b) $f(x) = 5x^2 - 8x - 2$

(c) $f(x) = 15x^3 + 8x^4 + 9x^2 + 7x - 8$

(d) $f(x) = 7x^5 + 8x^4 - 9x^3 + 6x^2 + 7\sqrt{x} - 1$

(e) $f(x) = 8x^4 - 7x - 8 + 10x^4$

(f) $f(x) = 4(x+3)^2$

(g) $f(x) = (107/x^{56}) + 8x^{47} - 5x^{31} - x^2 + (17/x) - 1$

(h) $f(x) = 67x^3 - 4x^5 + 30x - 9xx$

(i) $f(x) = -84.76 + 19.54x^2 - 35.08x$

(j) $f(x) = |9x^4 + 101x^3 - 9x^7 + 43x^2 - 8x + 502|$

(k) $f(x) = 2.1x^5 + 9.7x^4 - 9.8x^3 + 6.5x^2 + 7.0\sqrt{x} - 1.3$

(l) $f(x) = 84x^3 - 7xxxxx - 81x$

(m) $f(x) = -\pi^7x^6 - 6\pi x^4 - 2\pi^2x^3 + 49x^2 + \sqrt{\pi}x - \sqrt{6}$

(n) $f(x) = 107\sqrt{x^{56} + 8x^{47} - 5x^{31} - x^2 + (17/x) - 1}$

(o) $f(x) = (x+3)(x+2)(x+1)(x-1)(x-2)(x-3)$

(p) $f(x) = x^3(x-3)^2(x+5)^4$

11. Match each function given by a formula below with its graph. You should be able to do this *without* making your own graph of the function.

(a) $a(x) = x^3 - x$

(b) $b(x) = x^5 - 5x^3 + 4x$

(c) $c(x) = x^3 - 13x^2 + 54x - 72$

(d) $d(x) = x^4 + 17x^3 + 95x^2 + 199x + 120$

(e) $e(x) = (x-3)(x-2)(x-1)(x)(x+2)(x+4)$

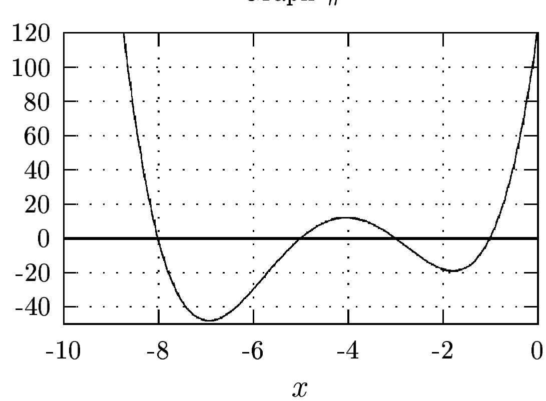
(f) $f(x) = x^4 - 5x^2 + 4$

(g) $g(x) = (x-3)(x-4)(x-1)$

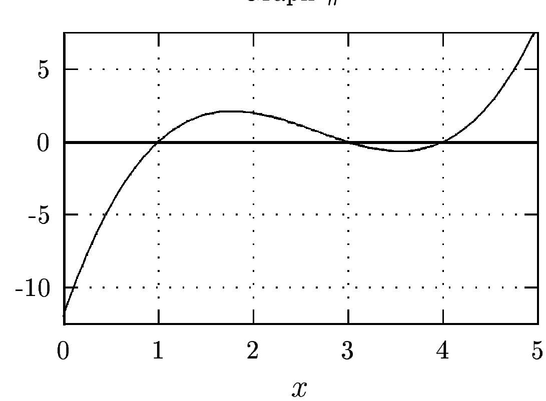
(h) $h(x) = (x-3)(x-2)(x-1)(x)(x+1)(x+2)(x+3)$

(i) $i(x) = x^7 - 3$

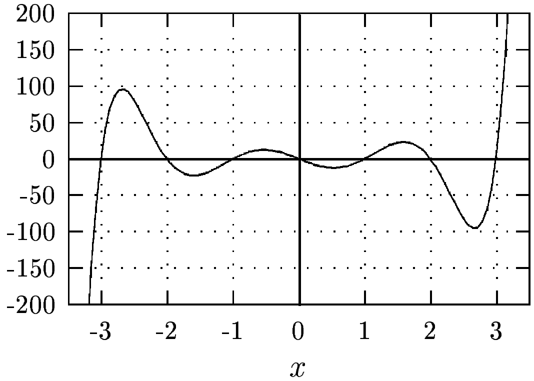
Graph #1



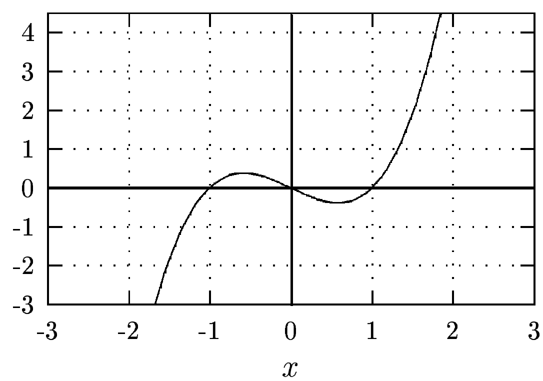
Graph #2



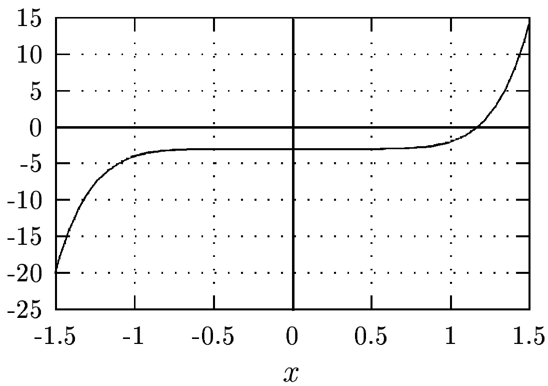
Graph #3



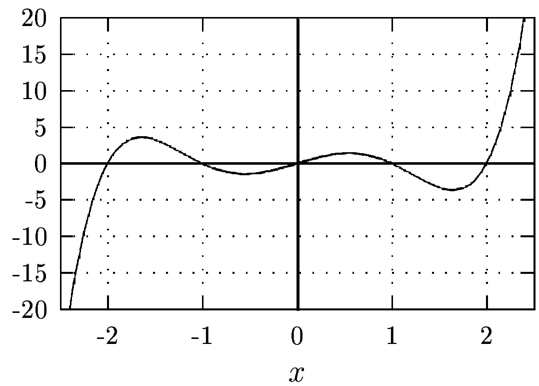
Graph #4



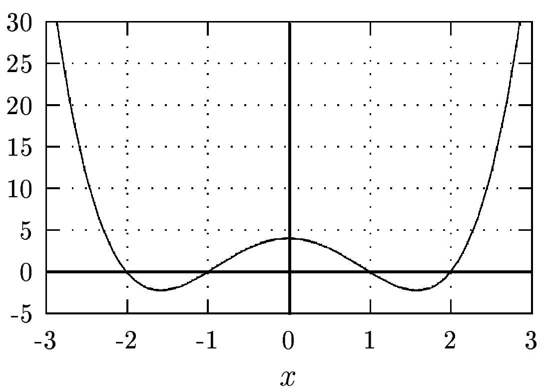
Graph #5



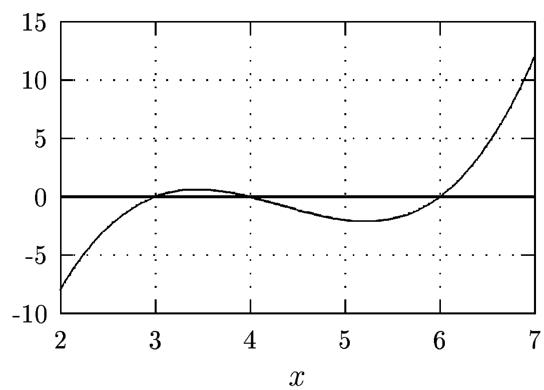
Graph #6

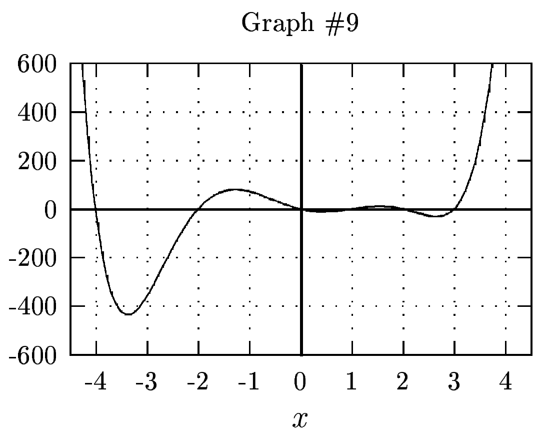


Graph #7



Graph #8



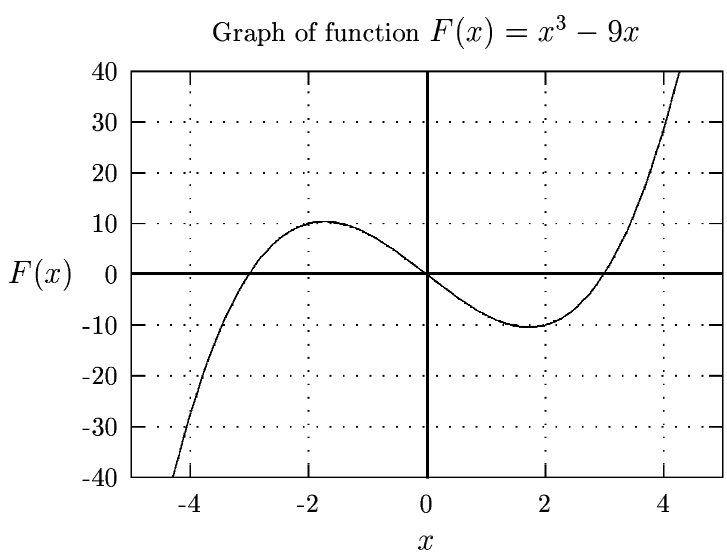


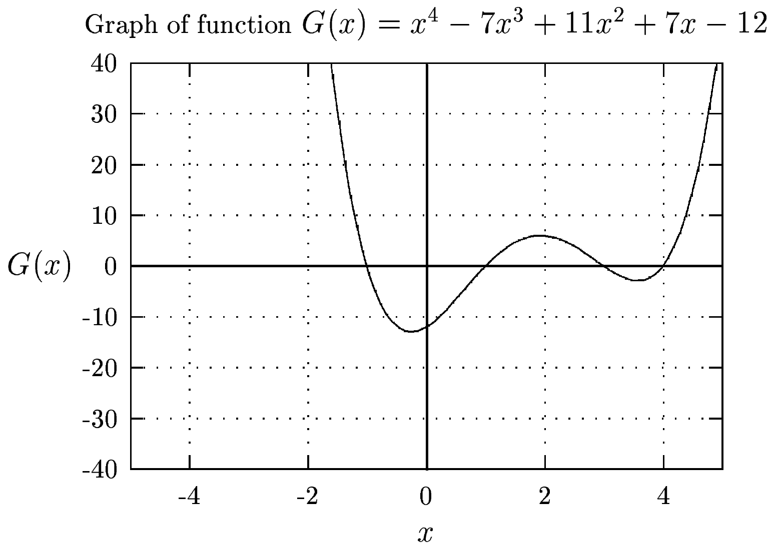
12. The first graph below is a plot of the function

$$\begin{aligned} F(x) &= x(x-3)(x+3) \\ &= x^3 - 9x, \end{aligned}$$

and the second graph is a plot of the function

$$\begin{aligned} G(x) &= (x-4)(x-3)(x-1)(x+1) \\ &= x^4 - 7x^3 + 11x^2 + 7x - 12. \end{aligned}$$





Match the following functions with the remaining graphs.

(a) $a(x) = x^3 - 9x - 10$

(b) $b(x) = (x+1)^3 - 9(x+1) = x^3 + 3x^2 - 6x - 8$

(c) $c(x) = (x-1)^3 - 9(x-1) - 20 = x^3 - 3x^2 - 6x - 12$

(d) $d(x) = F(x+2)$ (Note: F is the function defined above.)

(e) $e(x) = F(x) + 10$ (Note: F is the function defined above.)

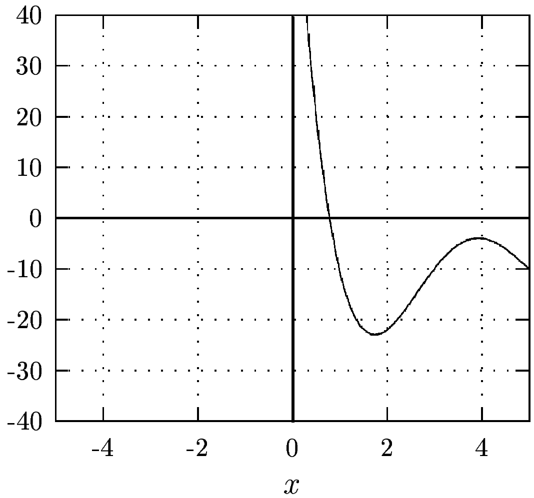
(f) $f(x) = (x-2)(x-1)(x+1)(x+3)$

(g) $g(x) = x^4 - 7x^3 + 11x^2 + 7x - 32$

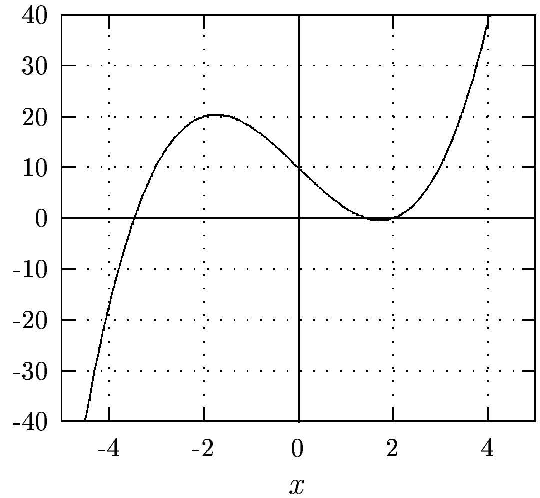
(h) $h(x) = G(x+3)$ (Note: G is the function defined above.)

(i) $i(x) = G(x-2) - 10$ (Note: G is the function defined above.)

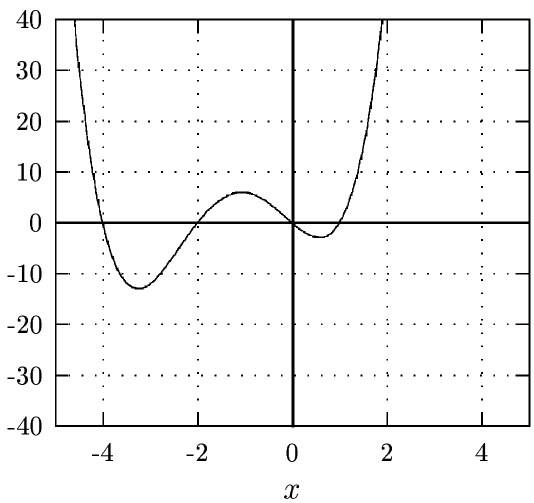
Graph #1



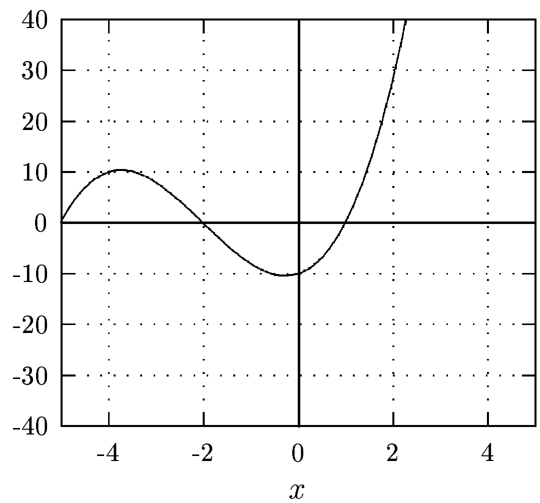
Graph #2



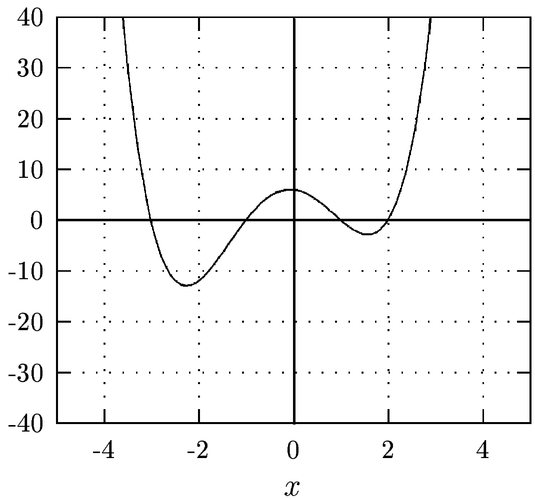
Graph #3



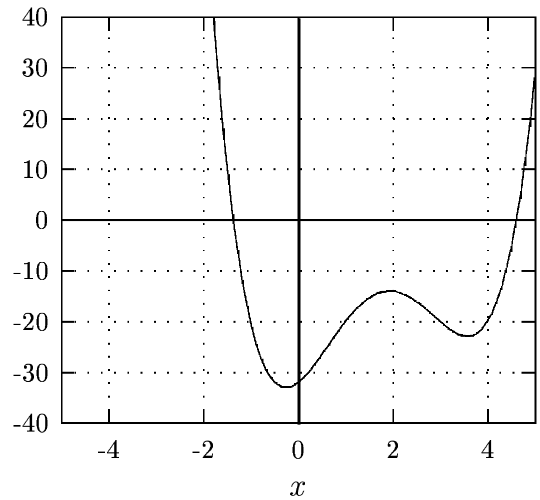
Graph #4



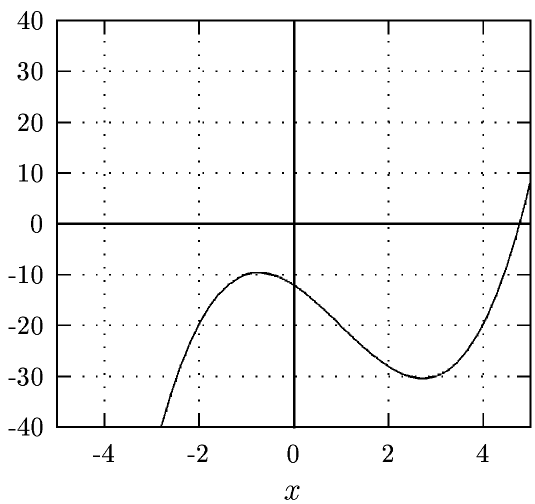
Graph #5



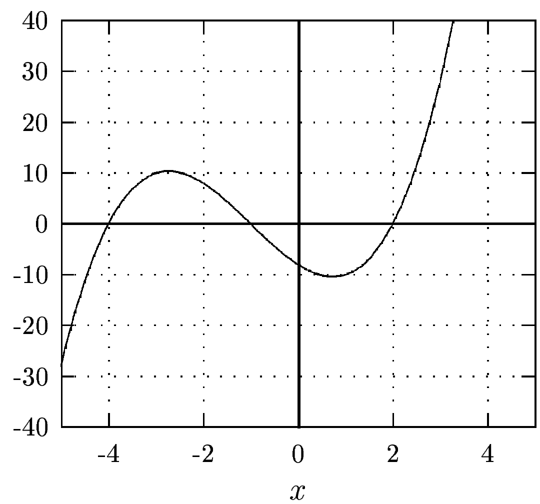
Graph #6

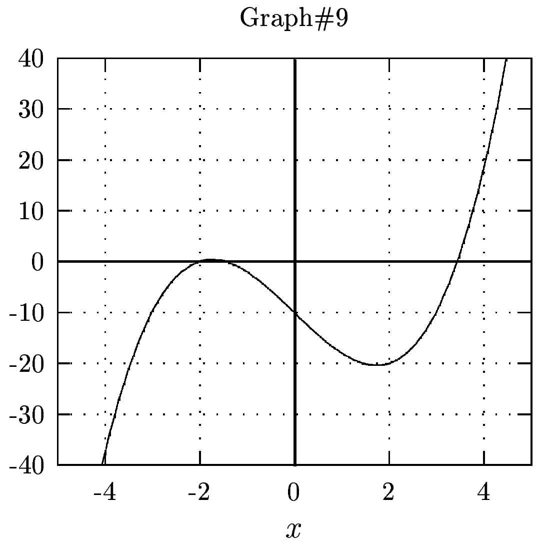


Graph #7



Graph #8





13. Give a formula for a polynomial P with the attributes described, if possible. If no such polynomial is possible, explain why not.

   (a) The roots of P are 2, 5, 7, and -3.
   (b) The roots of P are $\frac{1}{2}$ and 91.
   (c) The roots of P are all the integers from -5 to 3.
   (d) P has no roots and its y-intercept is 7.
   (e) P is quadratic and has exactly one root.
   (f) P is of degree 6 and has no roots.
   (g) P is of degree 5 and has no roots.
   (h) P is of even degree, its graph opens downward, and it has 4 roots.
   (i) P has y-intercept 3, is of even degree, opens downward, and has no roots.
   (j) P has exactly three extrema, all on the interval [-5, 5].

14. Give the intervals of increase of each polynomial graphed in the preceding exercise.

15. Give the roots of each of the following polynomials.

   (a) $P(x) = (x - 4)(x + 5)$
   (b) $P(x) = (x - 4)(x - 9)(x + 3)$

(c) $P(X) = (2x-1)(x-6)(x+7)$

(d) $P(x) = (x-107)(5x-4)(7x+1)$

(e) $P(x) = 4(x-1)^2(2x+3)^3)$

(f) $P(x) = x^2 - 5x + 6$

16. Give the formula for a polynomial with the set of roots given:

(a) 1, 2, 3

(b) -1, 4, 8

(c) 1/2, -4, 7 , 10

(d) 1/2, 3/8, 0

17. Evaluate the following expressions:

(a) $lim_{x\to+\infty}(4x^5 + 4x^2 + 3x - 9)$

(b) $lim_{x\to-\infty}(-2x^2 - 8x - 2)$

(c) $lim_{x\to+\infty}(6x^5 + 15x^3 + 9x^2)$

(d) $lim_{x\to-\infty}(-3x^5 + 8x^4 - 9)$

(e) $lim_{x\to-\infty}(8 - 7x - 8x^7)$

(f) $lim_{x\to+\infty}(-2x - 9)$

(g) $lim_{x\to-\infty}(-1.5x^2 - 80000x - 3)$

(h) $lim_{x\to+\infty}(5x^6 + 15x^5 + 4x^2)$

(i) $lim_{x\to-\infty}(x^5 + 8x^10 - 9x)$

(j) $lim_{x\to-\infty}(-2 - 70x + 80x^6)$

(k) $lim_{x\to-\infty}(-x^3 + 4x^2 + 3x - 9)$

(l) $lim_{x\to+\infty}(-2x^2 - 8x - 2)$

(m) $lim_{x\to-\infty}(5x^4 + 15x^3 + 9x^2)$

(n) $lim_{x\to+\infty}(-x^5 + 8x^4 - 9)$

(o) $lim_{x\to+\infty}(-8 - 7x - 3x^4)$

(p) $lim_{x\to-\infty}(-5x - 9)$

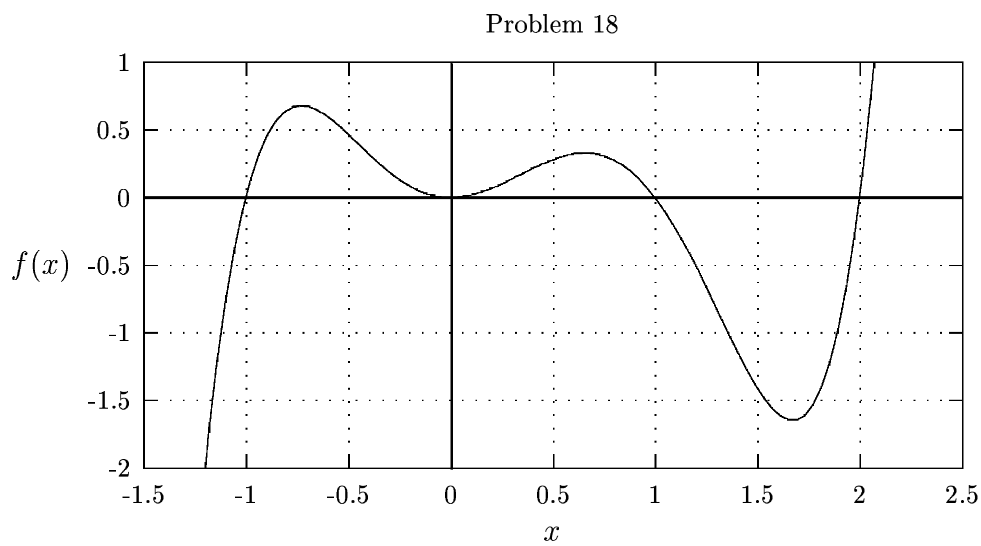
(q) $lim_{x\to+\infty}(-1.5x^2 - 4x - 3)$

(r) $lim_{x\to-\infty}(2x^6 + 15x^5 + 4x^2)$

(s) $lim_{x\to+\infty}(-x^5 + 8x^10 - 9x)$

(t) $lim_{x \to +\infty}(-2 - 7x + 3x^4)$

18. The graph of a fifth-degree polynomial is illustrated below.



(a) Is the aroc over the interval $[0, .5]$ positive, negative, or zero?

(b) Is the aroc over the interval $[0, 1]$ positive, negative, or zero?

(c) Is the aroc over the interval $[1, 1.5]$ positive, negative, or zero?

(d) Is the aroc over the interval $[-1, -.5]$ positive, negative, or zero?

(e) Is the aroc over the interval $[-1, 0]$ positive, negative, or zero?

(f) Give a numerical estimate of the a.r.o.c. over the interval $[-.5, 1]$.

19. Consider the fifth-degree polynomial $f(x) = x^5 + 5$.

(a) Calculate the aroc over the interval $[0, 1]$.

(b) Calculate the aroc over the interval $[1, 2]$.

(c) Calculate the aroc over the interval $[\sqrt{2}, \pi]$.

(d) Calculate the aroc over the interval $[-1, 0]$.