MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Chapter 7 Homework

1. Which of the following are rational functions?

$$a(x) = \frac{\sqrt{x}}{x^2 - 2}$$

$$b(x) = \frac{1}{x(x + \pi)}$$

$$c(z) = \frac{1}{z} - 2.3$$

$$d(x) = \frac{4x^{10} - 19x}{x}$$

$$e(r) = r + \frac{(r-1)(r+2)(r+6.1)}{(r-2)}$$

$$f(t) = \frac{1}{(2^t - 2)(t+2)}$$

$$g(x) = \frac{(5 + 6|x|)(3 - |x^3|)}{(x+2)}$$

$$h(y) = \frac{2}{y^3} + \sqrt{3.5}$$

$$i(z) = \frac{(3 + z^2 - z^3)^2}{(z^4 + 2)}$$

$$j(t) = 6t^3 - 7t^2 + 8t - 4$$

$$k(x) = \frac{7x^2 - 9}{x^3 + \frac{1}{x}}$$

$$l(z) = \frac{z^2 + 1}{\sqrt{8z - 1} + 4z}$$

$$m(t) = \frac{3t^6 + 4t - 3}{\sqrt{t}}$$

$$n(y) = \frac{(y-1)(y+2)}{(|y-1|}$$

$$o(x) = x + \frac{1}{x}$$

$$p(x) = \frac{2x}{x+1} + \frac{x+1}{2x}$$

$$q(x) = \frac{\sqrt{6}x^2 + 2}{x - 1}$$

$$r(t) = \frac{\frac{1}{2}}{t-2} + 8\sqrt{3t+2}$$

$$s(x) = \frac{1}{2^x} + \frac{x}{2x+1}$$

$$t(x) = \frac{x^2 - 1}{x} + \frac{2x^3}{x-3} + \frac{x}{\sqrt{x+1}}$$

2. Graph the rational function

$$F(x) = \frac{2}{5-x} + 3$$

by hand on a sheet of graph paper. Use the domain $[1, 9]$, plotting with special care near $x = 5$. (Plot at least 6 points in the interval $[4.7, 5.3]$)

3. Graph the rational function

$$R(x) = \frac{1}{x+2} - 3$$

by hand on a sheet of graph paper. Use the domain $[-5, 2]$, plotting with special care near $x = -2$. (Plot at least 6 points in the interval $[-2.3, -1.7]$)

4. Graph the rational function

$$f(x) = \frac{x}{3-x} + 2$$

by hand on a sheet of graph paper. Use the domain $[1, 4]$, plotting with special care near $x = 3$. (Plot at least 6 points in the interval $[2.7, 3.3]$)

5. Graph each of the following rational functions using a computer or graphing calculator. Sketch the results on graph paper, taking care to show clearly intercepts, asymptotes, limits as $x \to \pm\infty$, behavior near points not in the domain. Be aware of the limitations of the technology: there aren't really any spikes in the mathematical graph, just in some of the pictures. Find a good window. (Try starting a few units to the left of the left-most point not in the domain, and going a few units to the right of the right-most point not in the domain. Modify as needed.)

   (a)
   $$Q(x) = \frac{2}{6 - x}$$

   (b)
   $$R(x) = \frac{1}{x + 4} - 7$$

   (c)
   $$S(x) = \frac{-3}{3 - x} + 2$$

   (d)
   $$T(x) = \frac{x}{(x + 1)(x - 7)} + 3$$

   (e)
   $$U(x) = \frac{x - 4}{(x + 2)(x - 3)} - 3$$

   (f)
   $$V(x) = \frac{x^2 - 1}{x^2 + 1} + 2$$

6. For each of the following rational functions,

   - give the domain of the function
   - give the equation of each vertical asymptote (if any)
   - give the equation of each horizontal asymptote (if any)
   - give all x and y intercepts (if any)
   - give the limit of the function as $x \to \pm\infty$
   - for each value of x where the function has a vertical asymptote, give the left and right hand limits of the function as x approaches the asymptote.

   (a) $F(x) = \dfrac{3}{x}$

   (b) $G(x) = \dfrac{1}{x-7}$

   (c) $H(x) = \dfrac{4}{x+5}$

   (d) $J(x) = \dfrac{2}{3x+12}$

   (e) $L(x) = \dfrac{9}{(x-1)(x-3)}$

   (f) $M(x) = \dfrac{3}{x^2-x-12}$

   (g) $N(x) = \dfrac{5}{x^2-5x+6}$

   (h) $P(x) = \dfrac{1}{x^2+x-20}$

7. Consider the rational function

$$R(x) = \frac{x}{(x+5)(x-2.3)} + 10$$

   (a) Evaluate the following expressions:

   i. $\lim_{x\to -5^-} R(x)$
   ii. $\lim_{x\to -5^+} R(x)$
   iii. $\lim_{x\to 2.3^-} R(x)$

iv. $\lim_{x\to 2.3^+} R(x)$
v. $\lim_{x\to+\infty} R(x)$
vi. $\lim_{x\to-\infty} R(x)$

(b) Give the equation of each vertical asymptote.

(c) Give the equation of each horizontal asymptote.

8. Each of the functions *a*-*g* is plotted in the graphs labeled #1-#7. For each function, give the number of the graph on which the function is plotted.

(a) $a(x) = \dfrac{1}{x-3}$

(b) $b(x) = \dfrac{1}{x+3}$

(c) $c(x) = \dfrac{0.5}{x+2} - 5$

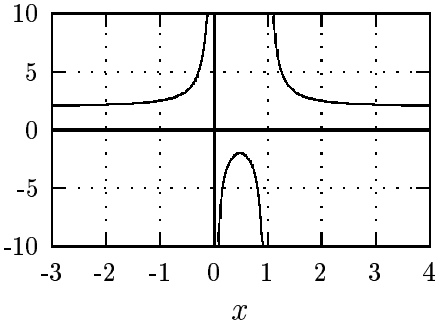
(d) $d(x) = \dfrac{3}{(x-2)(x+3)}$

(e) $e(x) = \dfrac{0.4}{x^2-7x+12}$

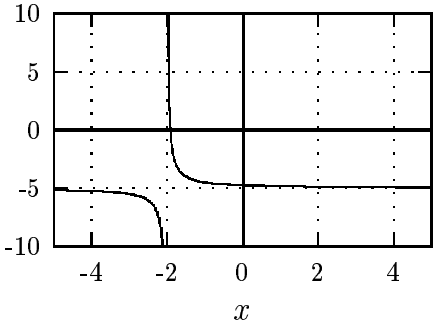
(f) $f(x) = \dfrac{1}{x^2-x} + 2$

(g) $g(x) = \dfrac{6}{(x-1)(x+4)}$

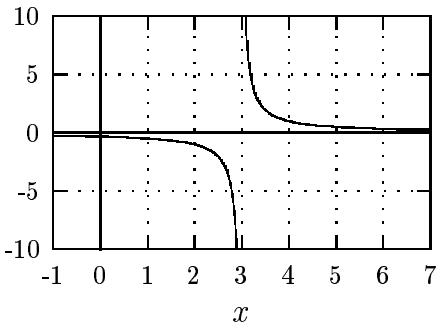
Graph #1



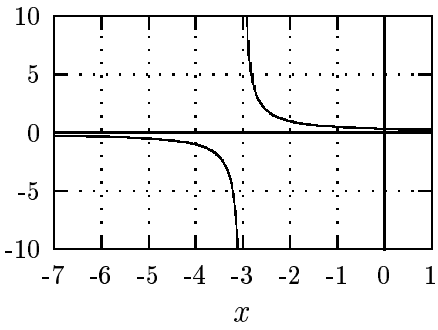
Graph #2



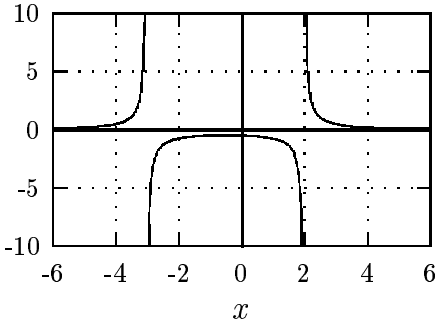
Graph #3



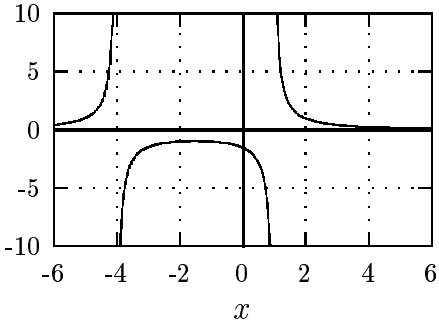
Graph #4



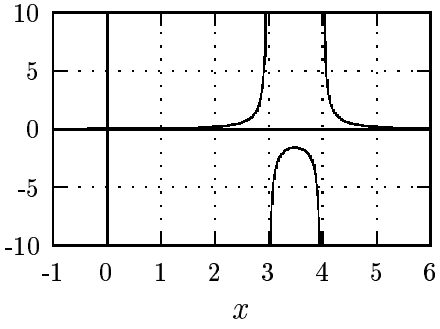
Graph #5



Graph #6

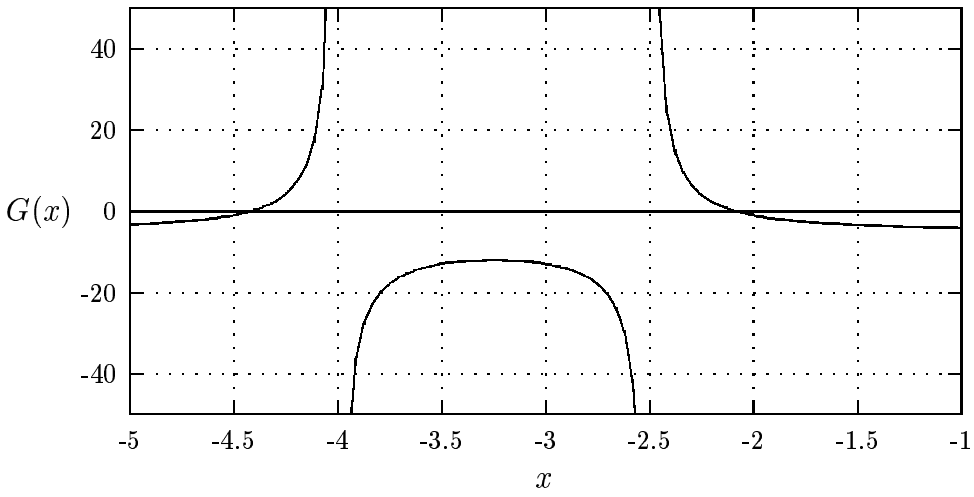


Graph #7



9. Assume the graph below is a fair representation of a rational function

Graph for Exercise 9



$G(x)$.

(a) Give all displayed intervals on which the function is increasing.

(b) Give all displayed intervals on which the function is decreasing.

(c) Give all displayed intervals on which $G(x) > 20$. (Estimate values of the endpoints of the intervals).

(d) Give all displayed intervals on which $G(x) > -20$. (Estimate values of the endpoints of the intervals).

(e) Determine whether G has any zeros. Estimate the values of any zeros from the graph.

(f) Does $G(x)$ appear to be undefined for any values of x? If so, which ones?

(g) From the graph find

i. $\lim_{x \to -4^-} G(x)$

ii. $\lim_{x \to -4^+} G(x)$

iii. $\lim_{x \to -2.5^-} G(x)$

iv. $\lim_{x \to -2.5^+} G(x)$

(h) Give the equation of each vertical asymptote. (You will have to estimate some values for your equation from the graph.)

(i) Give the equation of each horizontal asymptote. (You will have to estimate some values for your equation from the graph.)

10. Find the average rate of change of (aroc) of the function $f(x)$ given in Exercise 4 over each of the following intervals:

(a) $[-2, 0]$

(b) $[1, 2]$

(c) $[2, 2.5]$

(d) $[2.5, 2.8]$

(e) $[2.9990, 2.9999]$

(f) $[3.0001, 3.0002]$

(g) $[3.5, 4]$

(h) $[5, 10]$