MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Chapter 7
Class Exercises

1. Which of the following are rational functions?

$$a(x) = \frac{4x^3 - 3x + 5}{x - 1}$$

$$b(x) = \frac{|x^3 - 5x^2|}{6x^7 + 7x^6}$$

$$c(z) = \frac{(z-1)(z^2 + 6z + 1)}{10z^4 + 7}$$

$$d(x) = \frac{5x^2 + 2x + 3}{2}$$

$$e(r) = \frac{6r^4 - 7r + 3}{10\sqrt{r} + 4r^4 - 2r^2}$$

$$f(t) = \frac{t}{t - 2}$$

$$g(x) = \frac{\sqrt{6x^3 - 2x + 3}}{6x}$$

$$h(y) = \frac{3y + 6}{21y^6 - 3}$$

$$i(z) = \frac{1}{z}$$

$$j(t) = 6t^2$$

$$k(x) = \frac{1}{21x^3 + \sqrt{11}}$$

$$l(z) = \frac{6z^2 + \sqrt{5z} + 12}{4z}$$

$$m(t) = \frac{3t^6 + 4t - 3}{|t|}$$

$$n(y) = \frac{(y-1)(y+2)}{(y-1)}$$

$$o(x) = x + \frac{1}{x}$$

$$p(x) = \frac{2^x}{x+1}$$
$$q(x) = \frac{\sqrt{6}x^2+2}{x-1}$$
$$r(t) = \frac{5}{x-2} + 8$$
$$s(x) = \frac{1}{x} + \frac{x}{2x+1}$$
$$t(x) = \frac{1}{x} + \frac{2}{x-5} + \frac{x}{x+1}$$

2. Graph the rational function

$$f(x) = \frac{1}{x-2}$$

by hand on a sheet of graph paper. Use the domain $[-5, 9]$, plotting with special care near $x = 2$. (Plot at least 6 points in the interval $[1.7, 2.3]$.) What is special about the value $x = 2$ for this function?

3. Find a value of x for which the function $f(x) > 10,000,000$, where f is the function defined in Exercise 2.

4. Consider the function

$$g(x) = \frac{2}{(x-2)(x+3)}.$$

(a) Graph the function g using a computer or a graphing calculator. Make 2 graphs, the first with the domain $[-5, 5]$, and the second with the domain $[-10, 10]$.

(b) From the screen, estimate $g(-3)$, $g(0)$, $g(2)$, $g(5)$, and $g(10)$. Look at both graphs when determining your values (except for the last value, which doesn't appear on the first graph).

(c) Use algebraic techniques to find the exact value of $g(x)$ for the same inputs as in the previous problem.

(d) Do your answers in (b) and (c) agree? Should they?

(e) Does $g(x)$ equal exactly zero for any value of x? If so, what value? If not, how do you know?

(f) Is $g(x)$ undefined for any values of x? If so, which ones?

5. Consider the function

$$F(x) = \frac{2}{(x-1)(x+3)}.$$

(a) Graph F using a computer or a graphing calculator.

(b) Examine the graph and determine whether F has any zeroes. Estimate the values of any zeroes from the graph.

(c) Find an exact value for any zeroes of F using algebraic techniques.

(d) Do your answers for the two previous parts agree? Should they? Why or why not?

(e) Examine the graph and determine whether F has a y-intercept. If it does, estimate the value from the graph.

(f) If F has a y-intercept, find its exact value using algebraic techniques.

(g) Do your answers for the two previous parts agree? Should they?

(h) Do your answers for the two previous parts agree? Should they?

(i) What does the domain of F appear to be from the graph?

(j) What is the domain of F as determined by its formula? Give the largest possible domain, not the limited domain you see on a graph.

(k) What is the range of $F(x)$? (This is harder. Look at the graph.)

(l) Does $F(x)$ have any local maximum points? If so, what are the coordinates of the local maximum points? Use the graph on the screen for this. If you can think of any other way to answer this question, describe your method.

(m) Does $F(x)$ have any local minimum points? If so, what are the coordinates of the local minimum points? Use the graph on the screen for this. If you can think of any other way to answer this question, describe your method.

(n) Evaluate $F(x)$ at each integer for which this is possible, from -6 to 6. For the integers for which this is not possible, evaluate the function at values of x close to the integer on either side. Use at least three values of x on each side of the integers at which you can't evaluate F, and choose values of x that are not further than 0.1 from the integer.

(o) Evaluate the following expressions:

i. $\lim_{x\to 1^-} F(x)$
ii. $\lim_{x\to 1^+} F(x)$
iii. $\lim_{x\to 3^-} F(x)$
iv. $\lim_{x\to 3^+} F(x)$
v. $\lim_{x\to +\infty} F(x)$
vi. $\lim_{x\to -\infty} F(x)$

6. On what intervals is the function F given in Exercise 5 increasing?

7. Find a value of x for which the function $F(x) > 10,000,000$, where F is the function defined in Exercise 5.

8. Find a value of x for which the function $F(x) < 0.001$.

9. Repeat Exercise 5, but use the function

$$G(x) = \frac{x}{x^2 - 5x - 24}.$$

For part 5o, substitute the following:

(o) Evaluate the following expressions:

i. $\lim_{x\to 3^-} G(x)$
ii. $\lim_{x\to 3^+} G(x)$
iii. $\lim_{x\to -8^-} G(x)$
iv. $\lim_{x\to -8^+} G(x)$
v. $\lim_{x\to +\infty} G(x)$
vi. $\lim_{x\to -\infty} G(x)$

10. On what intervals is the function G in the previous problem increasing?

11. Find a value of x for which the function $G(x) > 10,000,000$.

12. Find a value of x for which the function $G(x) < 0.001$.

13. Repeat Exercise 5, but use the function

$$H(x) = \frac{x^2 - 5x + 6}{x - 1}.$$

For part 5o, substitute the following: Find the limit of the function as the value of x approaches 1 from the left and from the right.

14. On what intervals is the function H in the previous problem increasing?

15. Find a value of x for which the function $H(x) > 10,000,000$.

16. Find a value of x for which the function $H(x) < 0.001$.

17. Repeat Exercise 5, but use the function

$$I(x) = \frac{x^2 - x + 3}{x - 1}.$$

For part 5o, substitute the following: Find the limit of the function as the value of x approaches any point not in the domain of I from the left and from the right.

18. On what intervals is the function I increasing?

19. Find a value of x for which the function $I(x) > 10,000,000$.

20. Find a value of x for which the function $I(x) < 0.001$.

21. Repeat Exercise 5, but use the function

$$J(x) = \frac{x^2 - 9}{x}.$$

For part 5o, substitute the following: Find the limit of the function as the value of x approaches any point not in the domain of J from the left and from the right.

22. On what intervals is the function J increasing?

23. Find a value of x for which the function $J(x) > 10,000,000$.

24. Find a value of x for which the function $J(x) < 0.001$.

25. Repeat Exercise 5, but use the function

$$K(x) = \frac{x^2 - x - 6}{x + 1}.$$

For part 5o, substitute the following: Find the limit of the function as the value of x approaches any point not in the domain of K from the left and from the right.

26. On what intervals is the function K increasing?

27. Find a value of x for which the function $K(x) > 10,000,000$.

28. Find a value of x for which the function $K(x) < 0.001$.

29. Find the average rate of change (aroc) of $f(x)$ given in Exercise 2 over each of the following intervals:

    (a) $[-3, -2]$
    (b) $[-2, -1]$
    (c) $[-1, 0]$
    (d) $[0, 1]$
    (e) $[1, 1.5]$
    (f) $[1.5, 1.7]$
    (g) $[1.7, 1.9]$
    (h) $[1.9, 1.92]$
    (i) $[1.99, 1.992]$
    (j) $[1.99999, 1.999992]$

30. Write the formula of a rational function r satisfying the given conditions.

    (a) The function has poles at 1 and 7.
    (b) The function has poles at -3 and 2 and $\lim_{x\to\infty} r(x) = 0$.
    (c) The function has poles at -3 and 2 and $\lim_{x\to\infty} r(x) = 2$.
    (d) The function is not defined at 0 and 5 and $\lim_{x\to-\infty} r(x) = -4$.
    (e) The function has poles at -4 and -2 and zeroes 0 and -3.
    (f) The function has poles at -8 and -6 and zeroes -7 and -3.
    (g) The function has the asymptotes $x = 7$ and $y = -3$.

31. Each class group will be assigned a specific rational function. For the rational functions assigned to your group,

    (a) give the domain of the function,
    (b) give the equation of each vertical asymptote (if any),
    (c) give the equation of each horizontal asymptote (if any),
    (d) give all x and y intercepts (if any),

(e) give the limit of the function as $x \to \pm\infty$,

(f) for each value of x where the function has vertical asymptote, give the left and right hand limits of the function as x approaches the asymptote, and

(g) sketch the graph of the function on your transparency.