MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Unit 2— Functions Instructor's Notes

2.1 Key concepts

- Function:
  - definition
  - methods of representing a function
  - domain and range
  - graphs
  - functional notation
  - piecewise functions
- Interval notation
- Average rate of change

2.2 Summary

We start this unit with motion sensor work in which students produce a distance v. time graph on the computer or graphing calculator screen by the motion of their bodies in front of a sensor. We give them the definition of a function (intuitive version) and examples of functions presented in various ways; viz., by formulas, tables, graphs or verbal descriptions. We also introduce functional notation. Students work with several functions given in the various ways, mainly distance v. time functions.

We also introduce the idea of average rate of change of a function over an interval.

2.3 Class Outline

- Quiz on Unit 1
- Algebra push-ups
- Brief lecture on slope formula, using velocity = distance/time as main example

- Student work in groups duplicating templates with motion sensor, write up results
- Brief lecture on function concept, terminology, methods of presenting functions
- Students work on class exercises
- Journal (optional)
- Algebra homework: stock-related arithmetic, slopes of lines

2.4 Purposes

We introduce the concept of function with the usual terminology and notation, and present functions in different ways, so as to stress the idea of a correspondence. When functions are given only by algebraic formula, the focus is on the process of turning out answers — to evaluate for a specific value of the variable or to make a table for a graph. The multi-pronged approach, which of course is not original with us, is an effort to inculcate a less process-oriented concept of function, in the hope that later, when it is more fruitful to think of a function as a thing in itself (e.g., when considering inverses or derivatives), it will come more easily to students to do so. Piecewise functions are introduced here with the idea that they help in this endeavor. Interval notation is also introduced here (or rather, we hope, reviewed) because it's useful when working with piecewise functions.

2.5 Materials required

1. Computers or graphing calculators with motion sensors attached
2. Overhead transparencies with simple graphs on them to be placed over the screen and used as templates for students to match by appropriate movement. These should be numbered for reference

2.6 Advance preparation for the activities

There are motion sensors that can be attached to a computer via an interface, and there are some that can be attached to graphing calculators (e.g., TI).

We use computers when we have them, since the pictures are bigger and better. Graphing calculators with attachments are easier to carry around and set up.

The sensor measures the distance to the nearest object in front of it, and produces a distance versus time graph on the monitor. Some software can simultaneously show velocity v. time and acceleration v. time. There seems no point in presenting these to students at this stage, but they are useful in calculus.

We make simple piecewise-linear templates on transparencies which the students tape to the computer screen (or just set in front of it). The templates should be numbered and the axes should be marked on them to match the coordinate system shown on the screen. You should have graphs with increasing, decreasing and constant line segments on them. (Not every graph needs to have all of these). We use the time interval zero to ten seconds, and distance zero to three or four meters. It's good to have one or two more templates than groups of students.

Students need to walk backwards away from the sensor to keep an eye on the screen, and they need the space to do this, so you may need to move some furniture, or find a suitable place elsewhere if this is impossible.

2.7 What happens in our classroom

We divide the students into groups of about four, usually assigning the same groups as on the first day. We do a quick demonstration of moving to match the templates.. Then students take turns, so each one has at least one chance to walk in front of the sensor. Of the students not doing the moving, one should be assigned to start the data collection by clicking the mouse, one to describe in writing how the moving student moves to match the template, and one to see that the student doing the walking doesn't accidentally back into someone or something. Each group turns in its written description of the motion required for each template. (This is one reason for numbering the templates. The other is to help the groups keep track, since they need to trade the templates around so that each group does all.) They should describe the motion in terms of meters per second toward or away from the sensor.

We then discuss the slope formula, relating it to velocity, with reference to one of the motion sensor templates.

We introduce functional notation and do a few short examples, including one or two defined by a formula and one defined by a table. Then the students work on the in-class exercises. Before the students get to the piecewise functions in the exercises we need to do one or two of those. They always have trouble reading them.

Interval notation is something students used to know on entering, but we can't count on this, so we include a review of that and some related exercises. As usual students work in groups.

2.8 Discussion

The motion sensor exercises take up where the stock left off in terms of analyzing graphs. These exercises seem to give students a stronger understanding of what a graph does. Some students at first expect the line to stop when they stop. Now and again a student asked later on a test to describe the motion necessary to produce a graph will say that an interval of increase is produced by going uphill. We hope these rare cases illustrate how much the rest have learned, and what errant notions have been dispelled.

Many students enter our classes in passive mode, without much expectation of any connection with other students. The active work with motion sensors promotes connection, which helps learning. However, while some classes take to group work easily, others are slow and hesitant. This second get-up-and-move-around exercise expedites matters, we think, but some classes still need encouragement to work cooperatively. Doing so seems very efficient pedagogically, since it strongly promotes paying attention, verbalizing concerns and explanations, and forming human links related to mathematics, which in turn helps with making sense of it.

The first two of the in-class exercises are straightforward or only slightly twisted practice with functional notation. The graph-reading is intended as more practice with functional notation and interval notation.

Students do have trouble learning piecewise-function notation. (They also seem to find it easy to forget if it's not used for a while.) They need to see several examples.

The exercises on a walker's position are of course intended to build on the motion-sensor work, and to start students working with average rate of change, which we develop as the course goes on, in hopes it will help when the students come to derivatives in calculus.

(We do not bring up the fact that it's imipossible to duplicate the graphs on the template exactly, and since the students' efforts do not produce otherwise perfect matches, the issue doesn't arise. But of course it is impossible to change one's velocity instantaneously from, say, 1 m./sec. to -1m./sec. In fact in this course we havae not focused on any questions of error, except in discussion of the numerous unreliable decimal places the computers and graphing calculators show when purporting to provide the coordinates of a point.)