MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Unit 3— Quadratic Functions Instructor's Notes

3.1 Key concepts

- Recognition of the standard form of a quadratic: $ax^2 + bx + c$
- Graph of a quadratic: shape, vertex
- Vertex form of a quadratic and completing the square: $a(x - h)^2 + k$
- Roots of a quadratic:finding them by factoring, quadratic formula, graphically

3.2 Summary

We start the study of quadratics with a demonstration of a parabola: put a motion sensor at the head or foot of an inclined plane, give a large ball (e.g., bowling ball or basketball) a push up the plane, and see a parabola on the motion sensor screen as the ball goes up then down.

Students then graph quadratic functions, first by hand then using a graphing utility, experimenting with the latter to find the effects of changes in the various parameters, working first with the vertex form of a quadratic. They use the software to try to find the formula corresponding to a quadratic graph shown, by trial and error .

After that, students are given the standard formula for a quadratic expression, $ax^2 + bx + c$, and some practice in determining whether or not an expression is quadratic. They graph some quadratics and near-misses to see the differences in the graphs.

They also do some exercises involving average rate of change and parabolas.

3.3 Class Outline

- Algebra push-ups quiz
- Demonstration with ball on inclined plane
- Hand and graphing utility work with graphs of quadratics

- Brief lecture on standard form of a quadratic
- Exercises for groups in recognizing quadratics, algebraically and graphically
- Exercises for groups in finding the equation of a parabola answering a given description, if it exists
- Exercises for groups on average rate of change and parabolas
- Exercises with intervals
- Algebra homework: factoring, quadratic formula

3.4 Purposes

The demonstration of the ball going up then down the inclined plane is intended to give reality to the idea of a parabola.

The work with the graphing utility playing around with graphs and formulas is a vehicle for gaining insight into the nature of quadratics and the relationship between algebraic and geometric properties. Students should learn what the overall shape of a parabola is, how to locate roots and vertex graphically, what role the various parameters in the formula play.

We have found that many of our students do not take precision seriously in mathematical forms–if it looks something like the formula for a quadratic, they call it a quadratic (and the same with other forms). We have composed exercises that focus on forms.

This unit continues with the idea of average rate of change, relating it to quadratics.

3.5 Materials required

Computer or graphing calculator with motion sensor attachment, board or table about six feet long, with a rim along the edge to keep the ball from rolling off, something (a book or two will do) to prop up one end, and a large ball. A bowling ball is best, and a basket ball is okay. A large can also works, and for this the board doesn't need a rim. The largeness keeps the sensor beam, which fans out, from seeing past the ball.

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3.6 Advance preparation for the activities

Practice with the materials. You need to tap the ball hard enough to go up a ways but not over the top. You also need to practice timing with the sensor, and make sure that nothing in front of it is so near as to cause interference. (The beam from the sensor we use fans out at about a thirty-degree angle.)

3.7 What happens in our classroom

The demonstration of the ball on the inclined plane in front of the motion sensor is brief, but, we think, effective.

After the demonstration, the students should graph a quadratic by hand. It would save class time for students to do such a graph for homework before this class, although our experience is that this requires a pep talk and maybe a table of values to complete for the graph, because the idea of graphing a new type of equation (which this is to almost all the students) is distressing to most. Of course, one can do the graph oneself if pressed for time–but the students learn more doing it themselves.

When they graph using the computer or graphing calculator, students should make sketches of what they see on the screen. They may not want to do this. A brief introductory lecture on methodology, and some graph paper, possibly with coordinate axes (but not units) are desirable. You might want to include in the introductory lecture a remark about how you don't know that they're reading the graph correctly unless you get some feedback in the form of a sketch (or maybe just giving the roots and intercepts, and coordinates of the vertex). The next quiz might include a related question or two.

For recognition of quadratics from their equations we find it's best to go over a number of examples at the board, with student responses, before groups or individuals work on the exercises.

Students usually have a little trouble at first with the exercises on deciding whether or not there exists a parabola answering a given description and if so finding its formula, but get interested and figure it out. We have each group work on one such exercise and report its results to the class.

3.8 Discussion

We have found the brief demonstration with the ball on the inclined plane to be very effective. Students have told us afterwards that now they see where parabolas come from. In the past we have thrown chalk or whatever across the room and said the path is a parabola. We have no evidence that any insight resulted from this.

The bowling ball gives a much better parabola than does the basketball, which gives a slightly asymmetrical curve, perhaps because of properties of rubber. You wouldn't want anyone alert looking at it too closely–just from the middle distance.

Our students are often unenthusiastic about hand graphing, and, as we have mentioned before, slow at it. But when they use a graphing utility, many of them have a tendency to think the job consists of punching keys and getting a graph on the screen, period. At this stage of their education they seem to think of any analysis of what they see as a superfluity–they got a graph, didn't they?. However, the exercise on finding the vertex from the graph seems good training in reading a graph, and the conclusion is easy enough to reach. The exercise on graphing the quadratic-or-otherwise equations is another in which the visual results are reasonably simple and interesting.

If the students are not yet adept with the graphing utility, you'll need to monitor closely. One common trouble students have with software is failure to distinguish upper-case from lower-case letters when typing commands. Another and more challenging, issue is finding a good window. Our software, Maple, at least shows you a window that has some graph in it, but a graphing calculator may show nothing, and this is mysterious and frustrating to the neophyte.

The questions about the ball rolling on the inclined plane are intended to reinforce the demonstration of the parabola and analyzing graphs.

It was something of a shock to find out, after many years of giving the standard definition of a quadratic function and going on from there, that many of our students, while able to distinguish a quadratic from a higher degree polynomial, tend to think anything containing a power of two and no higher power is a quadratic. They don't take the form seriously. A fraction with a number in the numerator and a quadratic in the denominator looks like a quadratic to them. The exercises on recognizing quadratic functions algebraically and graphing quadratics and non-quadratics clears this up for most, but they need reminders later. (The issue of precision generally needs attention. Students are unconvinced that it matters to anyone but teachers.)

We continue with the idea of average rate of change, relating it to the parabola. Many students, perhaps most, continue to think of aroc in terms of the process of computing it. This is, after all, how they've learned to approach mathematics – as a collection of processes. (If you ask them to find the average rate of change of the line $y = 0.957x - 8.721$ over the interval $[-1.576, 8.015]$, they will use the formula for computing it, time and time again.)