MATH 161 — Precalculus[1]
Community College of Philadelphia

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Math 161 — Unit 4
Square root and absolute value functions and translations
Instructor's Notes

4.1 Key concepts

- Categorizing functions by type
- Preliminaries re absolute value and square root
- Absolute value functions: formulas, graphs
- Square root functions: formulas, graphs
- Translations
- Reflections
- Stretchings and shrinkings
- Domains and ranges from graphs
- Pythagorean theorem
- Distance formula

4.2 Summary

The unit starts with a refresher on square roots and absolute value, then proceeds to consideration of the absolute value and square root functions. These are used as a vehicle for work with translations and reflections, then stretching and shrinking of graphs. We continue with average rate of change, and throw in the Pythagorean Theorem and the distance formula. Students are given graphs from which to read domain and range and specified subsets thereof.

4.3 Class Outline

- Brief lecture, with transparencies, on absolute value function, with reference to piecewise functions.
- Brief lecture on square root function
- Class exercises on absolute value and square root functions
- Brief lecture on translations
- Class exercises on translations
- Brief lecture on reflections and stretching
- Class exercises on reflections and stretching
- Brief lecture on domain and range
- Class exercises on domain and range
- Brief lecture on Pythagorean Theorem and the distance formula
- Class exercises on Pythagorean Theorem and the distance formula

4.4 Purposes

Square root and absolute value functions are standard in the precalculus repertoire, and we introduce them in conjunction with translations, reflections and considerations of domain and range because these two functions each have a distinguished point (the corner on the absolute value function, the end point of the square root function) which is an aid in graphing. We also use these functions for exercises intended to promote awareness of considerations related to domain and range.

4.5 Materials and equipment required

1. An overhead projector
2. Transparencies:

(a) two with the square root function, one of these with the axes marked off in units, and a grid if possible, the other with the axes marked lightly if possible or not at all

(b) two of the same type for the absolute value function

(c) One with the line $y = x$ and one with the line $y = -x$, each with the part of the graph below the x-axis drawn lightly or with dashes

(d) One with a parabola on which your square root function can be imposed. To be really classy, you could have the parabola on unlabeled axes, and then a separate transparency with the coordinate system. If you want to try reflections in $y = x$ (not particularly recommended) you need equal scales on the two axes.

The overhead projector and transparencies above are for you, the instructor, to demonstrate the basic functions and their variants, but it's to the good if you can provide each group with a graph of each of the two basic functions on paper and a transparency copy of each, sans units on axes, so that they can do their own. (If not feasible, they can trace.)

4.6 Advance preparation

Making the transparencies described above and getting the overhead projector, if it isn't a fixture in your classroom.

4.7 What happens in our classroom

We start with a brief lecture on the absolute value function, which serves as a review of piecewise functions and the notation for them. Then we discuss graphing such functions when they are given by formulas.

The two transparencies, one with the line $y = x$ and one with the line $y = -x$, can be superimposed to produce the absolute value function, which shows up better if the extraneous parts of the original graph are faint. Students will be asked in the exercises to give, for various $f(x)$ and various c, intervals on which $f(x) > c$. The absolute value exercise can be treated as a warm-up for this. Then exercises.

The introduction to the square root function can be done by turning the parabola sideways. A possible lead-up to this is to show the parabola in its usual $y = x^2$ position, and ask what the square root of 9 is. Probably everyone will say 3. We show (or remind) them how to the input that gives 9 by starting at 9 on the y-axis, going to the graph and going down. We then note there's a second answer (giving credit of course if someone has already noted this). Then turn the parabola on its side, note that the result is not the graph of a function, opt for the positive half of it to get a function, and replace the parabola transparency with the half parabola. Then exercises.

The rules for vertical and horizontal translations seem made for using transparencies–just move the upper transparency around over the one that has the axes, units and grid. We do first vertical then horizontal translations, then both at once, and in each case we ask for the equations of the functions with the resulting graphs. Same with reflections in the x- or y-axis. It might be interesting to try reflections in the line $y = x$, but it would be easy to get bogged down in this. (To do reflections in $y = x$ you need transparencies with equal scales on the two axes.)

Then students can work on the exercises for computer or graphing calculator.

Most students find stretching and shrinking exercises difficult. We include some graphical exercises to help with this.

We continue with average rate of change, introducing exercises in which students find an interval on which the average rate of change is greater (or less) than a given number.

We also include the Pythagorean Theorem and the distance formula. If you do conics, that would be a good place to introduce these, since they're used there. We do not generally do a great deal with conics.

4.8 Discussion

The definition the absolute value function often gives trouble because piecewise notation gives trouble. Also, there is a strong tendency to regard an unadorned x as a positive number; after all, the usual way of getting the absolute value of a number is to drop the negative sign if there is one, and to do nothing otherwise. There are some class and homework exercises on absolute value intended to deal with this.

Students enjoy the activity of determining the formula of the function that results from moving the function graph, as illustrated with the transparency.

It's easier with a grid on the graph.

Stretching and shrinking are hard for most students. If you graph functions $f(x)$ and $3f(x)$ on the same coordinate system, people seem strongly predisposed to perceive the latter as narrower, not taller. This perception gets in the way. We have gone lightly over the topic in the past, but are trying some new exercises to work on this.