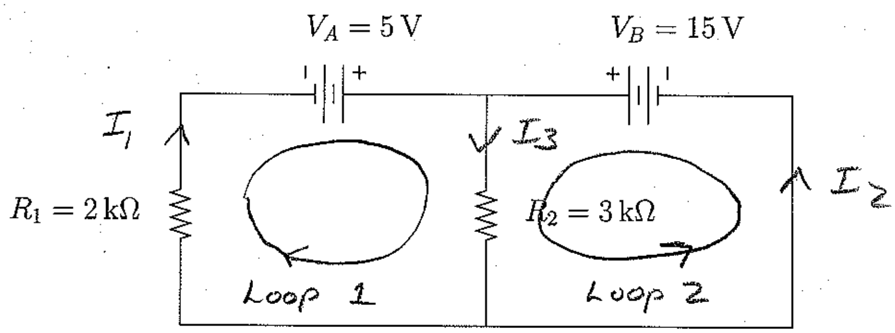
PHYS 235 — Exam #1
Tuesday, February 10, 2009

Name: KEY

1. Determine the current flowing through the $2\,\mathrm{k\Omega}$ resistor, the current flowing through the $3\,\mathrm{k\Omega}$ resistor, and the current flowing through the $15\,\mathrm{V}$ battery. **Be sure to indicate the direction of all three currents.**



Loop 1, KVL: $V_A - I_3 R_2 - I_1 R_1 = 0$

$\rightarrow 5 - 3I_3 - 2I_1 = 0$

Loop 2, KVL: $V_B - I_3 R_2 = 0$

$\rightarrow 15 - 3I_3 = 0$

$\Rightarrow I_3 = 5\,\mathrm{mA}$

Use this result here

$5 - 3\times5 - 2I_1 = 0$

$\Rightarrow I_1 = \frac{-10}{2} = -5\,\mathrm{mA}$

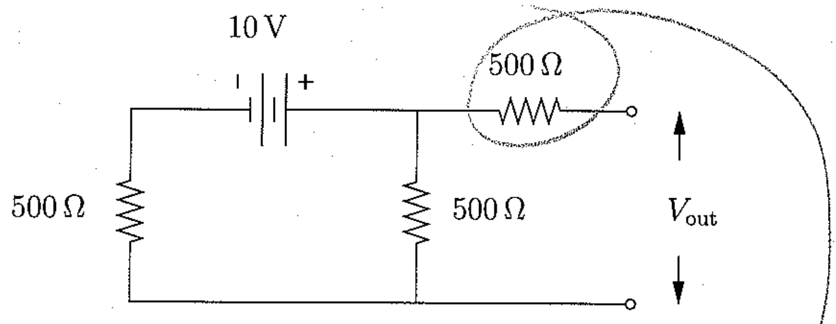
Oops! I guessed wrong direction for I_1; it must actually flow "down" through R_1.

KCL: $I_1 + I_2 = I_3 \Rightarrow I_2 = I_3 - I_1 = 10\,\mathrm{mA}$

2. (a) Determine the Thévenin equivalent voltage of the illustrated circuit.

   (b) Determine the Thévenin equivalent resistance of the illustrated circuit.

   (c) A 100 Ω resistor is attached across the illustrated output terminals of the circuit. Determine the current through the 100 Ω resistor.

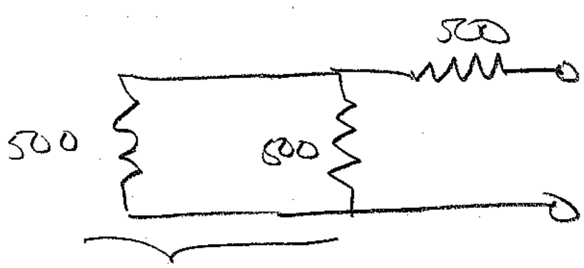


a) $V_{th} = V_{open\text{-}circuit}$

No current flows through this resistor under open-circuit conditions ⇒ no voltage drop across this resistor.

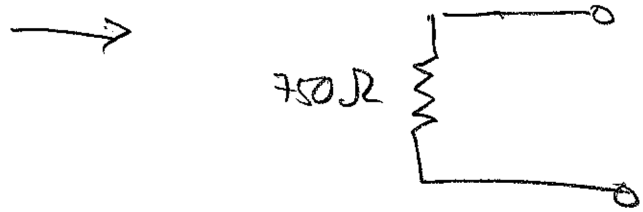
$$V_{th} = \frac{500\,\Omega}{500\,\Omega + 500\,\Omega} \times 10\text{ V} = 5\text{ V}$$

b) Method I: Short out battery and determine resistance between terminals



In parallel

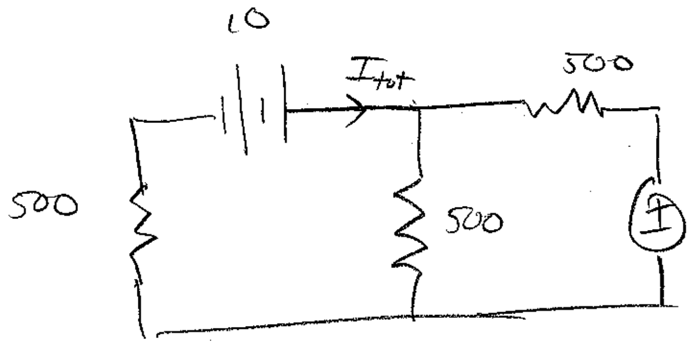
→



$R_{Th} = 750\,\Omega$

Method II:

$$R_{Th} = \frac{V_{Th}}{I_{short\text{-}circuit}}$$

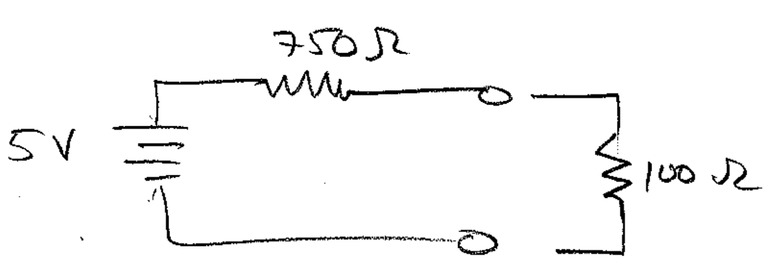


$$I_{tot} = \frac{10}{500+250} = \frac{10\,V}{750\,\Omega}$$

$$I_{s.c.} = \frac{1}{2} I_{tot} = \frac{5\,V}{750\,\Omega}$$

$$R_{Th} = \frac{V_{Th}}{I_{s.c.}} = \frac{5\,V}{5V/750\,\Omega} = 750\,\Omega$$

c) Equivalent circuit



$$I = \frac{5\,V}{750\,\Omega + 100\,\Omega}$$

$$= 5.88\,mA$$

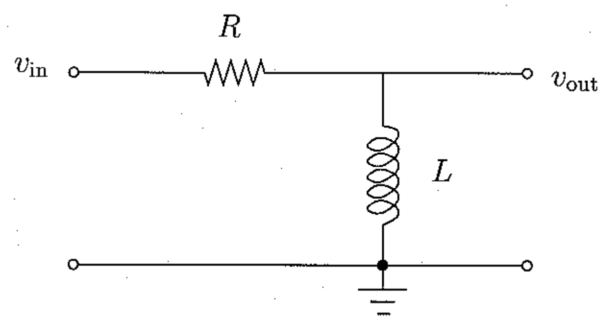
3. Consider the following circuit made of a resistor and an inductor.

   (a) Determine a formula giving the amplitude of the output as a function of frequency in terms or R and L.

   (b) If the value of the resistance is $R = 100\,\Omega$ and the value of the inductance is $L = 1/(20\pi) = 0.0159\,\text{H}$ and the input to the circuit (in volts) is

$$v_{\text{in}}(t) = 0.5\cos(2000\pi t),$$

   determine the **real** function $v_{\text{out}}(t)$ giving the output voltage as a function of time.



$$\tilde{V}_{out} = \frac{Z_L}{R+Z_L}\tilde{V}_{in} = \frac{j\omega L}{R+j\omega L}\tilde{V}_{in}$$

$$= \frac{j\omega L(R-j\omega L)}{R^2+\omega^2L^2}\tilde{V}_{in}$$

$$= \frac{\omega L\,(\omega L+jR)}{R^2+\omega^2L^2}\tilde{V}_{in}$$

$$= \frac{\omega L}{R^2+\omega^2L^2}\sqrt{R^2+\omega^2L^2}\;e^{j\tan^{-1}\frac{R}{\omega L}}\tilde{V}_{in}$$

$$= \frac{\omega L}{\sqrt{R^2+\omega^2L^2}}\,e^{j\tan^{-1}R/\omega L}\,\tilde{V}_{in}$$

$$= \frac{1}{\sqrt{1+\left(\frac{R}{\omega L}\right)^2}}\,e^{j\tan^{-1}R/\omega L}\,\tilde{V}_{in}$$

$$|\tilde{V}_{out}| = \frac{1}{\sqrt{1+\left(\frac{R}{\omega L}\right)^2}}\left(\tilde{V}_{in}\right)$$

In this problem $\omega = 2{,}000\,\pi$

& $\omega L = \frac{2{,}000\,\pi}{20\,\pi} = 100\,\Omega$

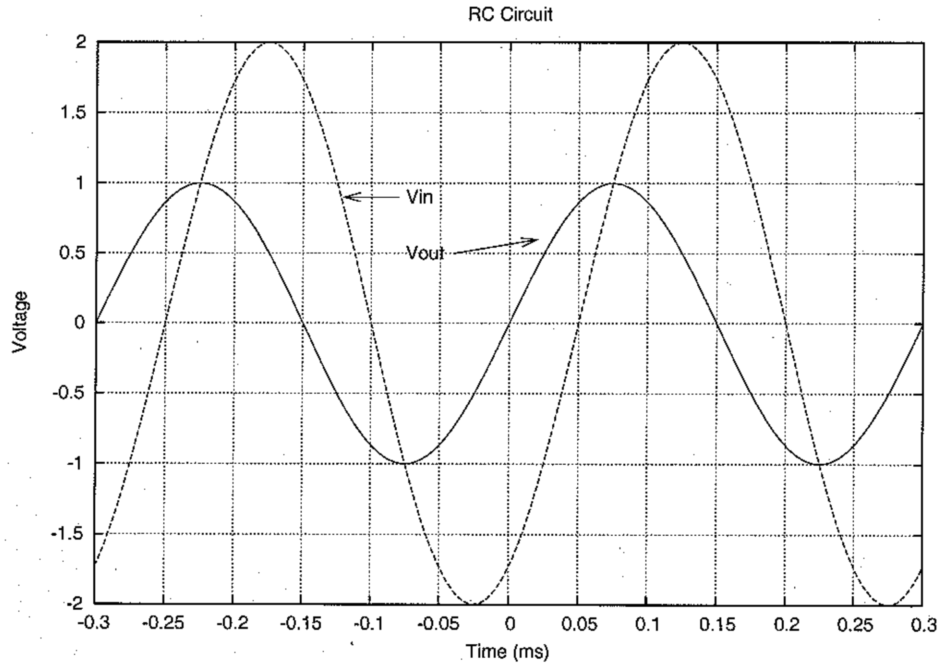
& $\left(\frac{R}{\omega L}\right) = \frac{100}{100} = 1$

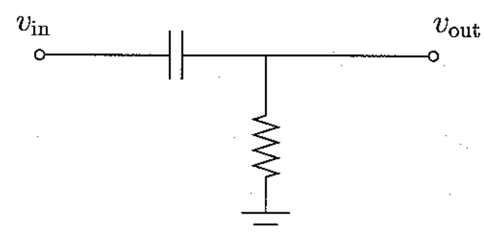
Phase shift of the signal is

$\tan^{-1}\frac{R}{\omega L} = \frac{\pi}{4}$

$$V_{out} = \frac{0.5}{\sqrt{2}}\cos\left(2000\pi t + \frac{\pi}{4}\right)$$

4. The graph below shows the input and output voltages for the illustrated RC circuit.





(a) Determine the phase shift of v_{out} relative to v_{in}.

(b) Sketch phasors in the complex plane corresponding to v_{in}, v_{out} and v_C.

(c) Determine from your phasor sketch the approximate amplitude of v_C, the voltage across the capacitor.

a) Time shift = 1 block

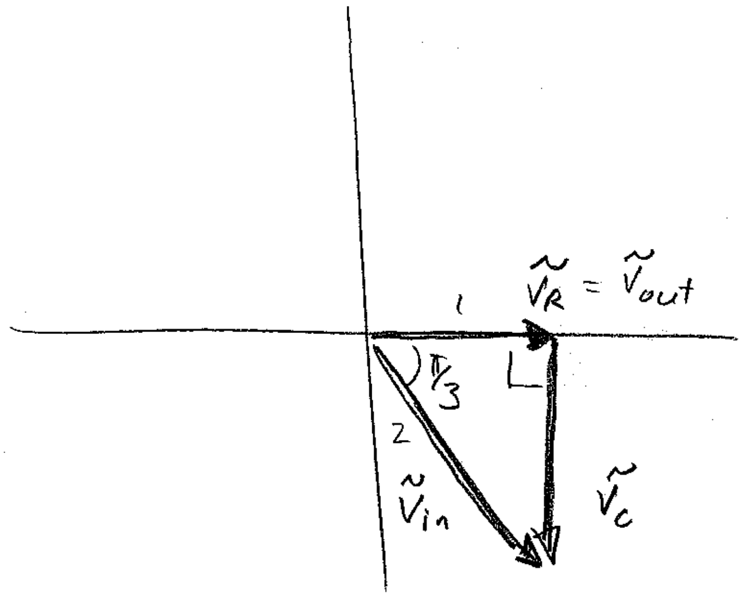
Period = 6 blocks

$\varphi = \frac{1}{6} \times 2\pi = \pi/3$ radians

V_{out} <u>leads</u> V_{in}

b) $\tilde{V}_{in} = \tilde{V}_c + \tilde{V}_R$

$\tilde{V}_R = \tilde{V}_{out}$



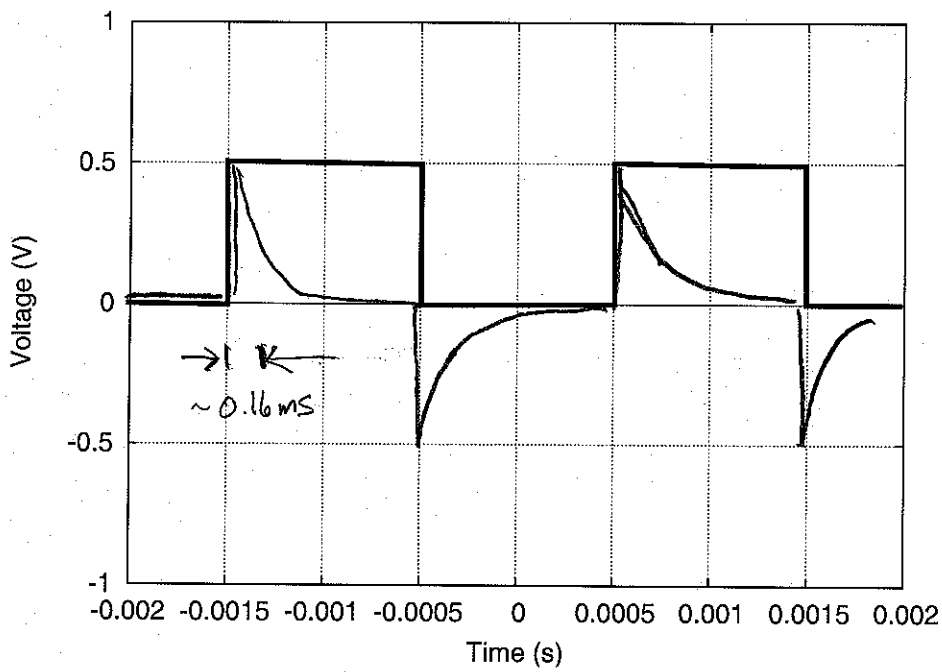
$$|V_c|^2 + |V_{out}|^2 = |V_{in}|^2$$

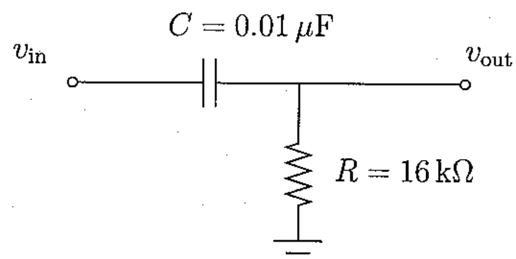
$$|V_c| = \sqrt{|V_{in}|^2 - |V_{out}|^2}$$

$$= \sqrt{4 - 1}$$

$$= \sqrt{3}$$

5. The graph below shows the input to the illustrated circuit. On the same graph sketch the output.





Capacitor will charge and discharge.
Vout is voltage across resistor, which means voltage will be proportional to current.
Vout will look like 

The only question is the time scale for charging/discharging. $RC = 16 \times 10^3 \times 10^{-8} = 1.6 \times 10^{-4}$ s
$= 0.16$ ms