

Numerical integration

```
In [1]: import scipy as sp
from scipy import integrate # not included in basic scipy

import matplotlib as mpl # As of July 2017 Bucknell computers use v. 2.x
import matplotlib.pyplot as plt

# Following is an Ipython magic command that puts figures in the notebook.
# For figures in separate windows, comment out following line and uncomment
# the next line
# Must come before defaults are changed.
%matplotlib notebook
#%matplotlib

# As of Aug. 2017 reverting to 1.x defaults.
# In 2.x text.ustex requires dvipng, texlive-latex-extra, and texlive-fonts-recommended
# which don't seem to be universal
# See https://stackoverflow.com/questions/38906356/error-running-matplotlib-in-latex-type
mpl.style.use('classic')

# M.L. modifications of matplotlib defaults using syntax of v.2.0
# More info at http://matplotlib.org/2.0.0/users/deflt_style_changes.html
# Changes can also be put in matplotlibrc file, or effected using mpl.rcParams[]
plt.rc('figure', figsize = (6, 4.5)) # Reduces overall size of figures
plt.rc('axes', labelsz=16, titlesz=14)
plt.rc('figure', autolayout = True) # Adjusts subplot parameters for new s.
```

Single variable

```
In [2]: def func1(x): # Continuous function
        return x**3

        def func2(x): # Discontinuous function
            if x < 2:
                return x**2
            if x > 2:
                return x**2

        def func3(x): # Function with a singularity
            return sp.sin(x-2)/(x-2)
```

```
In [3]: value, error = sp.integrate.quad(func1,1,2)
value, error
```

```
Out[3]: (3.7500000000000004, 4.1633363423443377e-14)
```

If there are discontinuities or singularities, quad will fail, eg.,

```
In [4]: value, error = sp.integrate.quad(func2,1,3)
value, error
```

```
-----
TypeError                                Traceback (most recent call last)
<ipython-input-4-228de3762d39> in <module>()
----> 1 value, error = sp.integrate.quad(func2,1,3)
      2 value, error

/usr/remote/anaconda-3.6/lib/python3.6/site-packages/scipy/integrate/quadpack.py in quad
quad(func, a, b, args, full_output, epsabs, epsrel, limit, points, weight, wvar, wopt
s, maxpl, limlst)
    321     if (weight is None):
    322         retval = _quad(func, a, b, args, full_output, epsabs, epsrel, limit,
--> 323                     points)
    324     else:
    325         retval = _quad_weight(func, a, b, args, full_output, epsabs, epsrel,

/usr/remote/anaconda-3.6/lib/python3.6/site-packages/scipy/integrate/quadpack.py in _
quad(func, a, b, args, full_output, epsabs, epsrel, limit, points)
    386     if points is None:
    387         if infbounds == 0:
--> 388             return _quadpack._qagse(func,a,b,args,full_output,epsabs,epsrel,l
imit)
    389     else:
    390         return _quadpack._qagi(func,bound,infbounds,args,full_output,eps
abs,epsrel,limit)

TypeError: must be real number, not NoneType
```

Specify troublesome points:

```
In [5]: value, error = sp.integrate.quad(func2,1,3,points=[2,])
value, error
```

```
Out[5]: (8.666666666666668, 9.621932880084691e-14)
```

```
In [6]: value, error = sp.integrate.quad(func3,0,4,points=[2,])
value, error
```

```
Out[6]: (3.210825953605389, 3.5647329017567276e-14)
```

Multivariable

Volume of a sphere I

$$V = \int_0^R \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} dz dy dx \equiv \int_0^R \int_{g(x)}^{h(x)} \int_{q(x,y)}^{r(x,y)} dz dy dx$$

With g , h , q , and r defined normally:

```
In [7]: def func4(z,y,x):    # ORDER OF ARGUMENTS IMPORTANT
        return 1

        def g(x):
            return 0

        def h(x):
            return sp.sqrt(xulim**2-x**2)

        def q(x,y):
            return 0

        def r(x,y):
            return sp.sqrt(xulim**2-x**2-y**2)
```

```
In [8]: xllim = 0
        xulim = 2
        value, error = sp.integrate.tplquad(func4, xllim, xulim, g, h, q, r)
        8*value, error, 4*sp.pi*xulim**3/3.
```

```
Out[8]: (33.51032163829113, 3.533511261366584e-10, 33.510321638291124)
```

With g , h , q , and r defined more concisely:

```
In [9]: g = lambda x: 0
        h = lambda x: sp.sqrt(xulim**2-x**2)
        q = lambda x,y: 0
        r = lambda x,y: sp.sqrt(xulim**2-x**2-y**2)
```

```
In [10]: xllim = 0
         xulim = 2
         value, error = sp.integrate.tplquad(func4, xllim, xulim, g, h, q, r)
         8*value, error, 4*sp.pi*xulim**3/3.
```

```
Out[10]: (33.51032163829113, 3.533511261366584e-10, 33.510321638291124)
```

Or even more concisely:

```
In [11]: xllim = 0
         xulim = 2
         value, error = sp.integrate.tplquad(func4, xllim, xulim, \
                                             lambda x:0, lambda x:sp.sqrt(xulim**2-x**2), \
                                             lambda x,y:0, lambda x,y:sp.sqrt(xulim**2-x**2-y**2),
         8*value, error, 4*sp.pi*xulim**3/3.
```

```
Out[11]: (33.51032163829113, 3.533511261366584e-10, 33.510321638291124)
```

Volume of a sphere I I

$$V = \int_0^{2\pi} \int_0^{\pi} \int_0^R r^2 \sin \theta dr d\theta d\phi$$

```
In [12]: def func5(phi,theta,r):
        return r**2*sp.sin(theta)
```

```
In [13]: rllim = 0
          rulim = 2
          value, error = sp.integrate.tplquad(func5, rllim, rulim, \
                                              lambda theta:0, lambda theta:sp.pi, \
                                              lambda theta,phi:0, lambda theta,phi:2.*sp.pi)
          value, error, 4*sp.pi*xulim**3/3.
```

```
Out[13]: (33.510321638291124, 5.556380318779972e-13, 33.510321638291124)
```

Version information

version_information is from J.R. Johansson (jrjohansson at gmail.com) See Introduction to scientific computing with Python: <http://nbviewer.jupyter.org/github/jrjohansson/scientific-python-lectures/blob/master/Lecture-0-Scientific-Computing-with-Python.ipynb> (<http://nbviewer.jupyter.org/github/jrjohansson/scientific-python-lectures/blob/master/Lecture-0-Scientific-Computing-with-Python.ipynb>) for more information and instructions for package installation. If version_information has been installed system wide (as it has been on linuxremotes), continue with next cell as written. If not, comment out top line in next cell and uncomment the second line.

```
In [14]: %load_ext version_information

          ##%install_ext http://raw.github.com/jrjohansson/version_information/master/version_info

Loading extensions from ~/.ipython/extensions is deprecated. We recommend managing ex
tensions like any other Python packages, in site-packages.
```

```
In [15]: version_information scipy, matplotlib
```

```
Out[15]:
```

Software	Version
Python	3.6.1 64bit [GCC 4.4.7 20120313 (Red Hat 4.4.7-1)]
IPython	6.1.0
OS	Linux 3.10.0 327.36.3.el7.x86_64 x86_64 with redhat 7.2 Maipo
scipy	0.19.1
matplotlib	2.0.2

Tue Aug 01 11:18:58 2017 EDT

```
In [ ]:
```