Light Propagation in Materials with a Negative Index of Refraction

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Atomic, Molecular, and Optical Physics Seminar
SUNY Stony Brook

July 21, 2010

“Common” Knowledge

Vacuum

$$E(z,t) = E_0 \cos(k^v z - \omega t)$$

$$v_{\text{ph}} = \frac{\omega}{k^v} = c$$

$$v_{\text{gr}} = \frac{d\omega}{dk} = c$$

Medium

$$E(z,t) = E_0 \cos(nk^v z - \omega t)$$

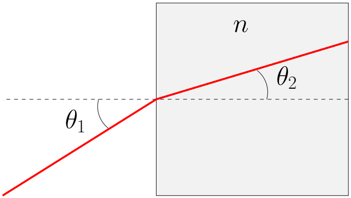
$$v_{\text{ph}} = \frac{\omega}{k} = \frac{\omega}{nk^v} = \frac{c}{n}$$

$$v_{\text{gr}} = \frac{d\omega}{dk} = \frac{c}{n + \omega \frac{dn}{dk}}$$

"Common" Knowledge

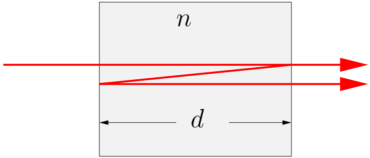
Refraction

$$\sin\theta_1 = n\sin\theta_2$$



Interference

$$\Delta\phi_{\text{path}} = 2\pi\frac{n\Delta r}{\lambda^v}$$



$$\Delta r = 2d$$

"UnCommon" Knowledge

Vacuum

$$E(z,t) = E_0 \cos(k^v z - \omega t)$$

$$v_{\text{ph}} = \frac{\omega}{k} = c$$

$$v_{\text{gr}} = \frac{d\omega}{dk} = c$$

Medium with $n < 0$

$$E(z,t) = E_0 \cos(n k^v z - \omega t)$$

$$v_{\text{ph}} = \frac{\omega}{k} = \frac{\omega}{n k^v} = \frac{c}{n}$$

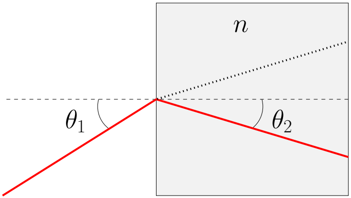
$$v_{\text{gr}} = \frac{d\omega}{dk} = \frac{c}{n + \omega \frac{dn}{dk}}$$

?

"UnCommon" Knowledge

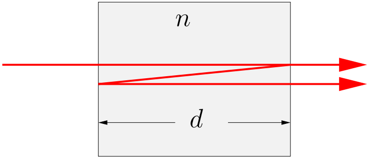
Refraction

$$\sin\theta_1 = n\sin\theta_2$$



Interference

$$\Delta\phi_{\text{path}} = 2\pi\frac{n\Delta r}{\lambda^v} < 0$$



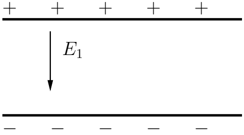
$\Delta r = 2d$

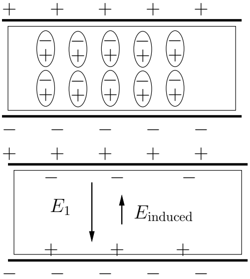
Whence $n < 0$?

Material with:

$$\begin{aligned} \epsilon &< 0 \\ \mu &< 0 \end{aligned}$$

Electric Fields in Materials

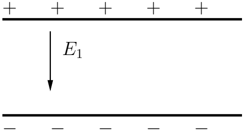


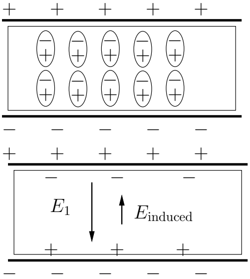


$$E_{\mathsf{tot}} = E_1 - E_{\mathsf{induced}}(E_{\mathsf{tot}}) = E_1 - \alpha E_{\mathsf{tot}}$$

$$E_{\mathsf{tot}} = \frac{1}{1+\alpha} E_1$$

Electric Fields in Materials





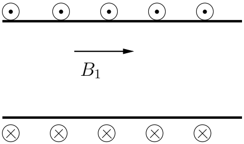
$$E_{\mathsf{tot}} = E_1 - E_{\mathsf{induced}}(E_{\mathsf{tot}}) = E_1 - \alpha E_{\mathsf{tot}}$$

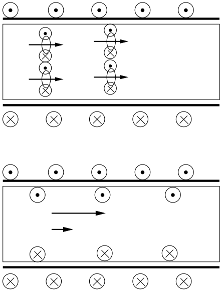
$$E_{\mathsf{tot}} = \frac{1}{1+\alpha} E_1$$

Total Field determined from free charge distribution:

$$\vec{E} = \frac{1}{\epsilon}\vec{D}$$

Magnetic Fields in Materials





Total field determined from field from free currents and field from magnetization.

$$\vec{B} = \mu \vec{H}$$

Metamaterials

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$\longrightarrow \quad \oint \vec{D} \cdot d\vec{A} = Q_{\text{enc, free}}$$

$$\longrightarrow \quad \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc, free}}}{\epsilon}$$

Origin of n (Mathematical)

Vacuum

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{and} \quad \nabla \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial z^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow E(z,t) = E_0 \sin(\sqrt{\epsilon_0 \mu_0}\, \omega\, z \pm \omega t)$$

$$\text{speed} = \omega / k^v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \equiv c$$

Origin of n (Mathematical)

Material

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{and} \quad \nabla\times\vec{B} = \epsilon\,\mu\;\frac{\partial\vec{E}}{\partial t}$$

$$\Rightarrow \frac{\partial^2 E_x}{\partial z^2} = \epsilon\,\mu\;\frac{\partial^2 E_x}{\partial t^2}$$

$$\Rightarrow E(z,t) = E_0 \sin(\sqrt{\epsilon\,\mu}\;\omega z \pm \omega t)$$

$$\text{speed} = \omega/k = \frac{1}{\sqrt{\epsilon\,\mu}} = \frac{1}{\sqrt{\epsilon_0\mu_0}}\sqrt{\frac{\epsilon_0\mu_0}{\epsilon\,\mu}} \equiv \frac{c}{n}$$

Whence a negative n?

Material with:

$$\begin{aligned} \epsilon &< 0 \\ \mu &< 0 \end{aligned}$$

- Argument 1: For $\epsilon < 0$ and $\mu < 0$, mathematical reasons for choosing the negative square root in $\sqrt{\epsilon\mu}$.
- Argument 2: Use physical considerations and standard boundary-value conditions. (Consider entire wave in vacuum and medium.)

Familiar Results (Vacuum)

$$\vec{E}_{\text{tr}} = \vec{E}_0 e^{i(\vec{k}^v \cdot \vec{r} - \omega t)}$$

$$\vec{B}_{\text{tr}} = \sqrt{\epsilon_0 \mu_0}\, E_0 e^{i(\vec{k}^v \cdot \vec{r} - \omega t)} \left(\hat{\mathbf{k}} \times \hat{\mathbf{E}}_\mathbf{0} \right)$$

$$= \frac{1}{c}\, E_0 e^{i(\vec{k}^v \cdot \vec{r} - \omega t)} \left(\hat{\mathbf{k}} \times \hat{\mathbf{E}}_\mathbf{0} \right)$$

$$\vec{E}_{\text{sw}} = \vec{E}_0 \cos(\vec{k}^v \cdot \vec{r}) \cos(\omega t)$$

$$\vec{B}_{\text{sw}} = \sqrt{\epsilon_0 \mu_0}\, E_0 \sin(\vec{k}^v \cdot \vec{r}) \sin(\omega t) \left(\hat{\mathbf{k}} \times \hat{\mathbf{E}}_\mathbf{0} \right)$$

$$= \frac{1}{c}\, E_0 \sin(\vec{k}^v \cdot \vec{r}) \sin(\omega t) \left(\hat{\mathbf{k}} \times \hat{\mathbf{E}}_\mathbf{0} \right)$$

$$\vec{S} = \frac{1}{\mu_0} \left(\vec{E} \times \vec{B} \right)$$

Familiar Results (Medium)

$$\vec{E}_{\text{tr}} = \vec{E}_0 e^{i(n\vec{k}^v \cdot \vec{r} - \omega t)}$$

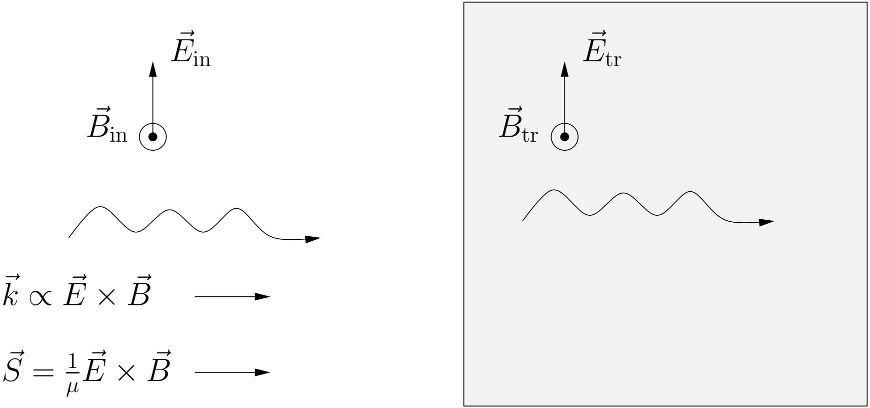
$$\vec{B}_{\text{tr}} = \frac{\sqrt{\epsilon\mu}}{c} E_0 e^{i(n\vec{k}^v \cdot \vec{r} - \omega t)} \left(\hat{\mathbf{k}} \times \hat{\mathbf{E}}_\mathbf{0}\right),$$

$$\vec{E}_{\text{sw}} = \vec{E}_0 \cos(n\vec{k}^v \cdot \vec{r}) \cos(\omega t)$$

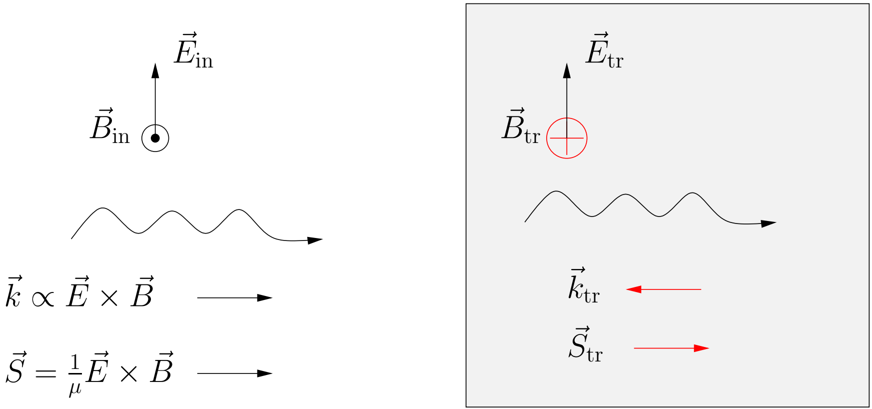
$$\vec{B}_{\text{sw}} = \frac{\sqrt{\epsilon\mu}}{c} E_0 \sin(n\vec{k}^v \cdot \vec{r}) \sin(\omega t) \left(\hat{\mathbf{k}} \times \hat{\mathbf{E}}_\mathbf{0}\right),$$

$$\vec{S} = \left(\vec{E} \times \vec{H}\right) = \frac{1}{\mu}\left(\vec{E} \times \vec{B}\right),$$

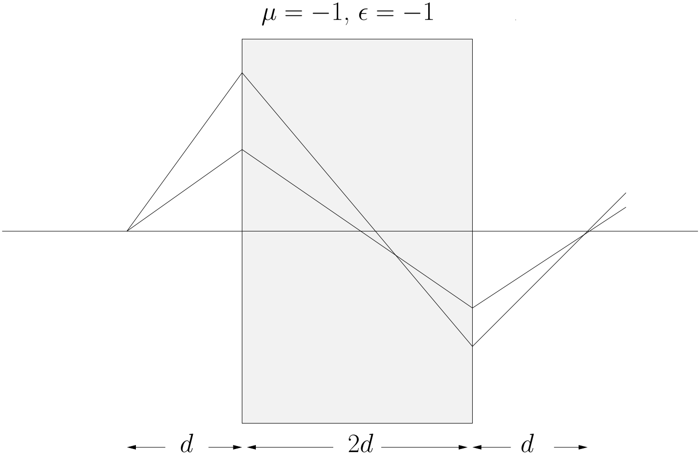
Consequences of $\epsilon < 0$, $\mu < 0$



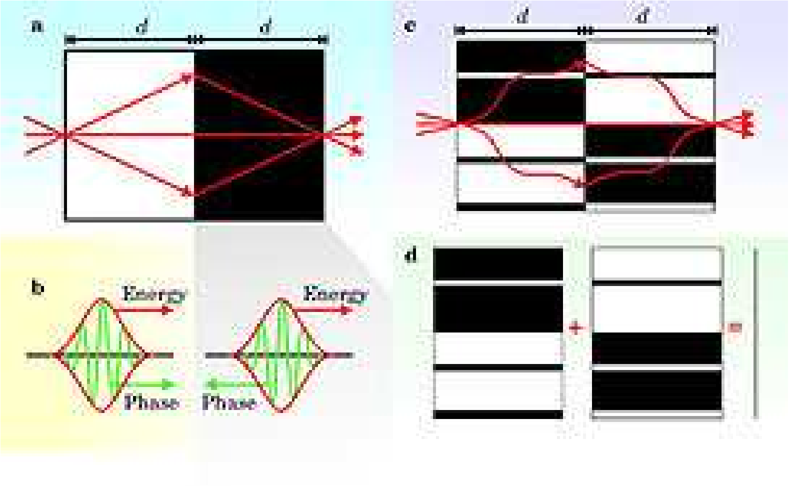
Consequences of $\epsilon < 0$, $\mu < 0$



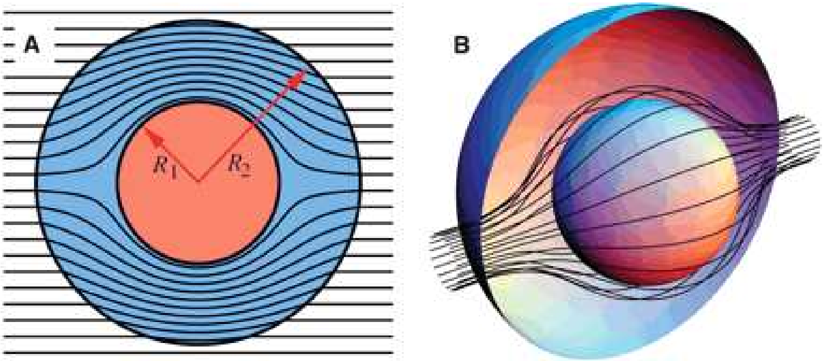
Lensing



“Negative Space”



Cloaking



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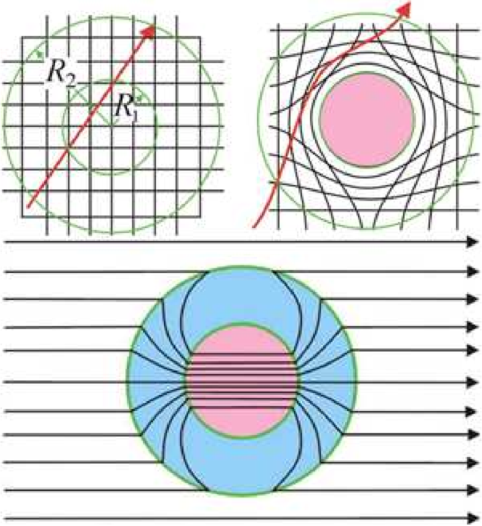
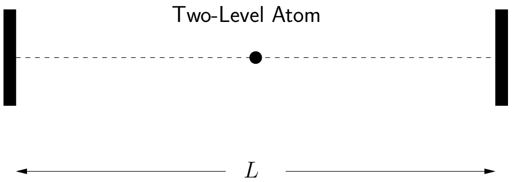


Model for quantum system



Model Features:

- "Modes of the universe" (1-D); Quantized standing wave modes
- Multiple modes (5001) $\rightarrow$ quasi-continuum
- Spontan. emission via interaction with multiple empty modes.
- Schrödinger picture.
- $\longrightarrow$ "Localized" photons.

Model for quantum system

Basis States:

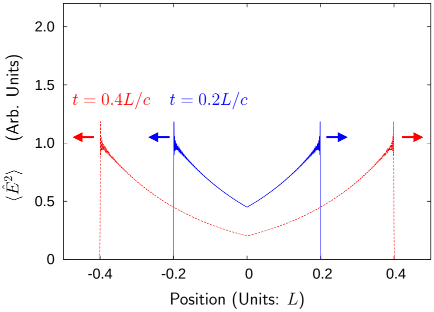
$|e;\emptyset\rangle$: atom excited, no photons
$|g;1_m\rangle$: atom in g.s., 1 photon (mode m)

Initial State: $|\psi(0)\rangle = |e;\emptyset\rangle$

Time-Dependent State:

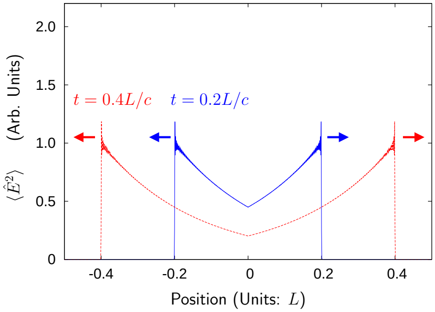
$$|\psi(t)\rangle = c(t)|e;\emptyset\rangle + + \sum_m b_m(t)|g;1_m\rangle$$

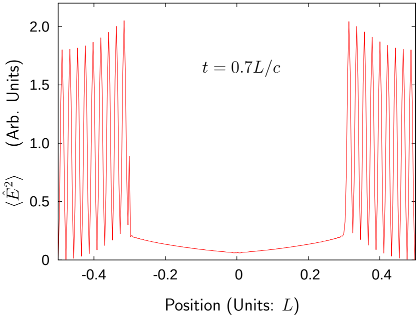
Expectation Value of Intensity



No phase information

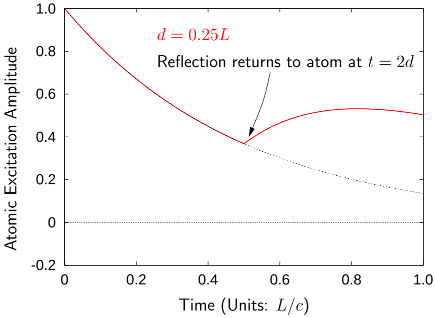
Expectation Value of Intensity



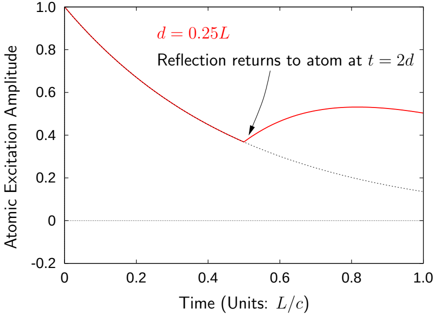


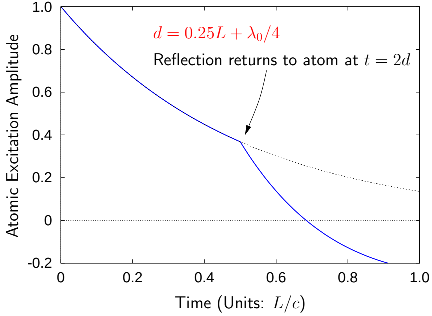
Phase information revealed in interference

Atomic Excitation Amplitude



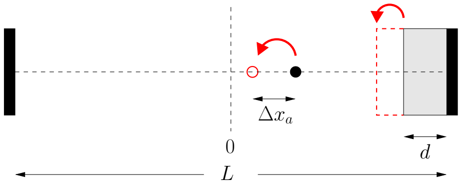
Atomic Excitation Amplitude



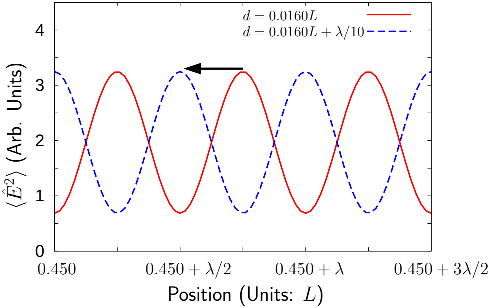


Phase information revealed in interaction with source

Manifestation of Accumulation of Negative Phase



Manifestation of Accumulation of Negative Phase



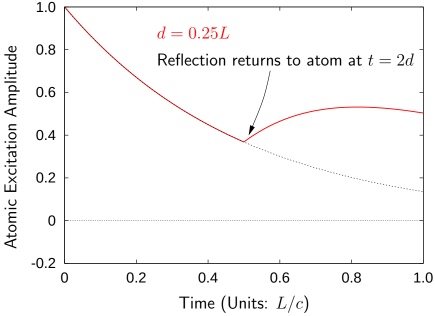
Change thickness by $\Delta d = \frac{1}{10}\lambda_{\text{vac}}$

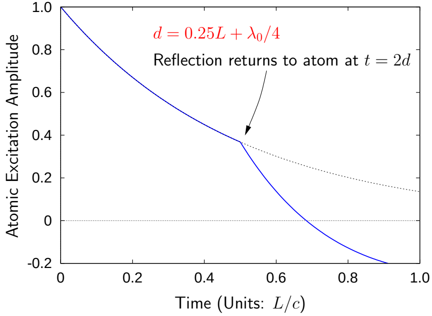
$\Rightarrow$ Optical path length changes: $\Delta(\text{OPL}) = 2(n-1)\Delta d$

$\Rightarrow$ Shift in standing wave: $\Delta z = (n-1)\Delta d = -\frac{1}{4}\lambda_{\text{vac}}$

Standing wave shifts to the left.

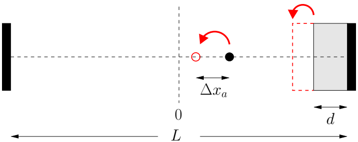
Atomic Excitation Amplitude





Phase information revealed in interaction with source

Manifestation of Accumulation of Negative Phase



Move atom to left by Δx_a

$\Rightarrow$ Optical path length increases:

$$\Delta(\text{OPL}) = 2\Delta x_a$$

$\Rightarrow$ Change in re-excitation of source atom

To compensate for the increased optical path, increase thickness of medium:

$$\Delta d = \frac{\Delta x_a}{(n-1)}$$