PHYS 333 — Exam #3
Due Monday November 24, 2014

This is a closed-book, timed, take-home exam. You are to complete the exam in one sitting of not more than 90 minutes. The only materials you are to use are the equations and information pages provided with this exam.

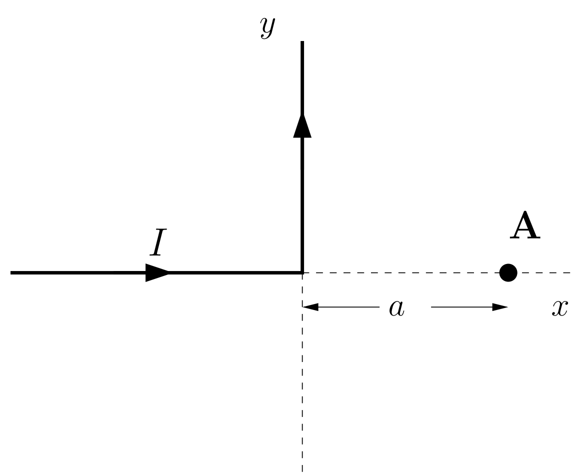
If you do not understand a problem statement, or if you have other questions, contact me and "stop the clock" until your question is answered.

- Name:
- Date:
- Location:
- Start time:
- Finish time:

Please sign below affirming that you have followed the conditions set for this exam.

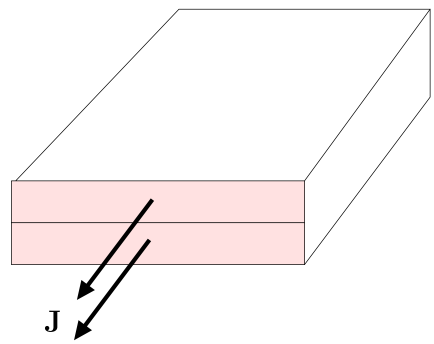
Signature:________________________

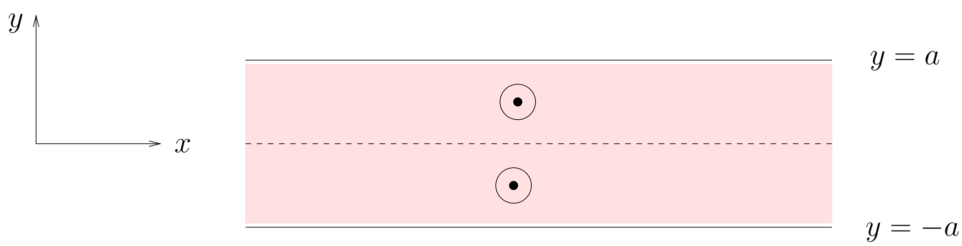
1. (20 pts) A long wire carrying a current I is bent so that it has a right-angle turn. The wire lies in the x-y plane as illustrated, with the right-angle turn located at the origin. Determine an integral expression for the magnetic field at the indicated point **A**, a distance a away from the origin on the x-axis. Your integral should be expressed in terms of x, y, z, $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$, a, I and physical constants. You do not need to evaluate the integral. (I'm not asking for the final expression for the field — you may be able to give me that without the integral expression. I want to see your integral.)



2. (20 pts) Consider the illustrated section of an infinite slab carrying a time-independent volume current density that depends on the absolute value of the y-coordinate: $\mathbf{J} = c|y|\,\hat{\mathbf{z}}$, where c is a constant.

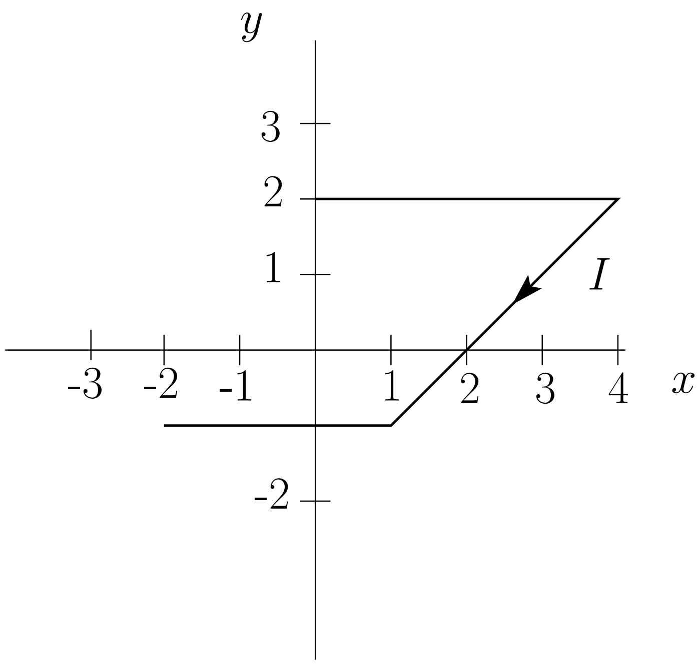
Determine the magnetic field everywhere.





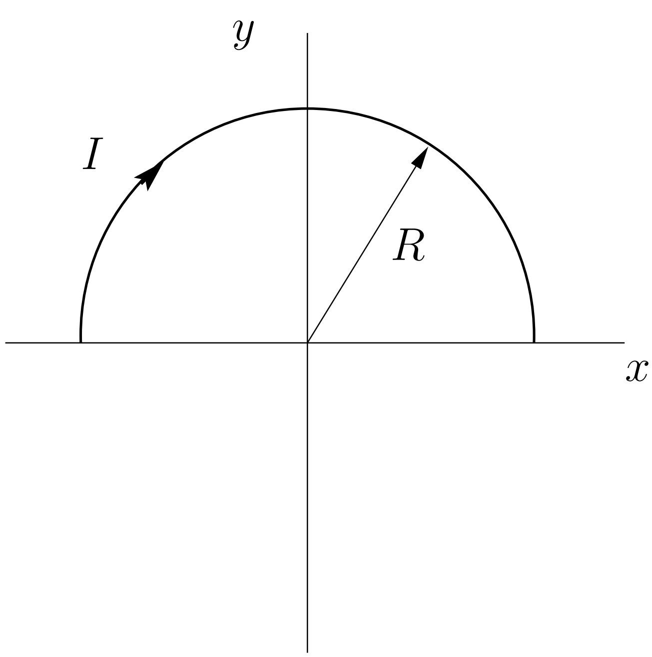
3. (20 pts.) Determine the force on the illustrated segments of current carrying wire in the indicated magnetic fields. All currents lie in the x-y plane.

(a)



$\mathbf{B} = B\,\hat{\mathbf{z}}$

(b)



$\mathbf{B} = B\,\hat{\mathbf{z}}$

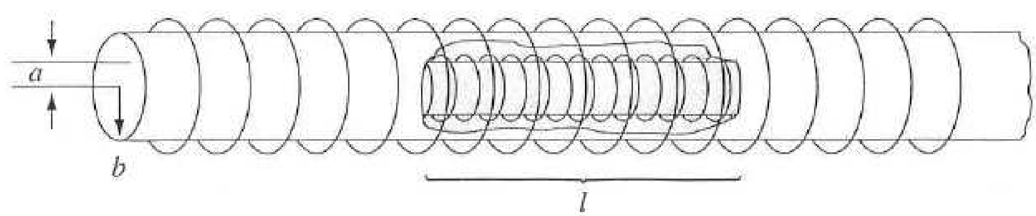
4. (20 pts) Write down the electric and magnetic fields for a monochromatic plane wave propagating in free space with magnetic field amplitude B_0, wavelength λ, and phase angle zero, for the following cases:

   (a) a wave traveling in the negative z direction and polarized in the y direction;

   (b) traveling in the direction from the origin to the point $(1, 1, 0)$, with polarization parallel to the x-y plane.

   Your answer should be expressed in terms of B_0, λ, Cartesian coordinates, Cartesian unit vectors, and physical constants.

5. (20 pts) A *short* solenoid (length l and radius a with n_1 turns per unit length lies within a very long ($\sim$infinite) solenoid (radius b with n_2 turns per unit length) as illustrated. The two solenoids are co-axial. As a reminder, the magnetic field of an infinite solenoid carrying a current I is constant within the solenoid, with a magnitude $B_{\text{solenoid}} = \mu_0 n I$, and zero outside the solenoid.



(a) Determine the mutual inductance of the two solenoids.

(b) There is a time-dependent current $I_1(t) = I_0(1 - \alpha t)$ in the inner short solenoid that circulates counter-clockwise as viewed from the left. Determine the emf induced in the outer (infinite solenoid).

(c) In what direction will any induced currents flow in the outer solenoid? Clockwise or counter-clockwise, as viewed from the left?

Equations

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \longleftrightarrow \frac{\mu_0}{4\pi} \int \frac{\mathbf{K}(\mathbf{r}') \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} da' \longleftrightarrow \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} d\tau'$$

$$\mathbf{I} = \lambda \mathbf{v}$$

$$\mathbf{K} \equiv \frac{d\mathbf{I}}{dl_\perp} \longrightarrow \sigma \mathbf{v}$$

$$\mathbf{J} \equiv \frac{d\mathbf{I}}{da_\perp} \longrightarrow \rho \mathbf{v} \longleftrightarrow I = \int \mathbf{J} \cdot d\mathbf{a}$$

$$\mathbf{F} = q\left[\mathbf{E} + (\mathbf{v} \times \mathbf{B})\right]$$

$$\mathbf{F}_{\mathrm{mag}} = \int I(d\mathbf{l} \times \mathbf{B}) \longleftrightarrow \int (\mathbf{K} \times \mathbf{B})\, da \longleftrightarrow \int (\mathbf{J} \times \mathbf{B})\, d\tau$$

$$\mathbf{m} = I \int d\mathbf{a}$$

$$\mathbf{B}_{\mathrm{dip}}(\mathbf{r}) = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\, \hat{\mathbf{r}} + \sin\theta\, \hat{\boldsymbol{\theta}}\right) = \frac{\mu_0}{4\pi r^3}[3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}]$$

$$\mathbf{N} = \mathbf{m} \times \mathbf{B}$$

$$\mathbf{F} = \boldsymbol{\nabla}(\mathbf{m} \cdot \mathbf{B})$$

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\mathcal{E} \equiv \oint \mathbf{f}_s \cdot d\mathbf{l} \longrightarrow \oint (\mathbf{f}_{\mathrm{elec}} + \mathbf{f}_{\mathrm{mag}}) \cdot d\mathbf{l}$$

$$\Phi \equiv \int \mathbf{B} \cdot d\mathbf{a}$$

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$$\oint \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\int \mathbf{B}\cdot d\mathbf{a} \longleftrightarrow \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 \left(I_{\text{enc}} + \epsilon_0 \frac{\partial}{\partial t}\int \mathbf{E}\cdot d\mathbf{a} \right) \longleftrightarrow \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

$$\mathbf{J}_d \equiv \epsilon_0 \frac{\partial \mathrm{E}}{\partial t}$$

$$\Phi = LI \qquad \text{and} \qquad \Phi = MI$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$W = \frac{1}{2}LI^2$$

$$\nabla^2 \mathbf{E} = \mu_0\epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad \nabla^2 \mathbf{B} = \mu_0\epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Traveling Plane Wave Solutions:

$$\mathbf{E}(\mathbf{r},t) = E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)\,\hat{\mathbf{n}} \qquad \mathbf{B}(\mathbf{r},t) = \frac{E_0}{c}\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta)\,(\hat{\mathbf{k}} \times \hat{\mathbf{n}}) = \frac{1}{c}\,\hat{\mathbf{k}} \times \mathbf{E}(\mathbf{r},t)$$

$$\tilde{\mathbf{E}}(\mathbf{r},t) = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}\,\hat{\mathbf{n}} \qquad \tilde{\mathbf{B}}(\mathbf{r},t) = \frac{1}{c}\,\tilde{E}_0\, e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}\,(\hat{\mathbf{k}} \times \hat{\mathbf{n}}) = \frac{1}{c}\,\hat{\mathbf{k}} \times \tilde{\mathbf{E}}(\mathbf{r},t)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

$$V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r}')}{\mathscr{r}}\, d\tau' \longrightarrow \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\mathbf{r}')}{\mathscr{r}}\, da' \longrightarrow \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\mathbf{r}')}{\mathscr{r}}\, dl' \longrightarrow \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{\mathscr{r}_i}$$

$$\mathbf{E} = -\vec{\nabla} V$$

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}} \qquad \& \qquad \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = -\frac{1}{\epsilon_0}\sigma$$

$$W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j=1, j>i}^{n} \frac{q_i q_j}{\mathscr{r}_{ij}} = \frac{1}{8\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \frac{q_i q_j}{\mathscr{r}_{ij}}$$

$$W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\mathbf{r}_i) \longrightarrow \frac{1}{2} \int \rho V \, d\tau$$

$$W = \frac{\epsilon_0}{2} \int_{\text{all space}} E^2 \, d\tau$$

$$C \equiv \frac{Q}{\Delta V}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$V(\mathbf{r}) = V_{\text{monopole}}(\mathbf{r}) + V_{\text{dipole}}(\mathbf{r}) + \cdots$$

$$V_{\text{monopole}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

$$V_{\text{dipole}} = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\, \hat{\mathbf{r}} + \sin\theta\, \hat{\theta}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \left[3(\mathbf{p} \cdot \hat{\mathbf{r}})\, \hat{\mathbf{r}} - \mathbf{p}\right]$$

$$Q = \sum_{i=1}^{n} q_i \longrightarrow \int \rho(\mathbf{r}')\, d\tau'$$

$$\mathbf{p} = \sum_{i=1}^{n} q_i \mathbf{r}_i \longrightarrow \int \mathbf{r}' \rho(\mathbf{r}')\, d\tau'$$

$$\mathbf{N} = \mathbf{p} \times \mathbf{E}$$

$$\mathbf{F} = (\mathbf{p} \cdot \nabla)\mathbf{E}$$

$$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$$

$$\rho_b = -\nabla \cdot \mathbf{P}$$

$$\rho = \rho_b + \rho_f$$

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$$

$$\nabla \cdot \mathbf{D} = \rho_f \longrightarrow \oint \mathbf{D} \cdot d\mathbf{a} = Q_{f_{\text{enc}}}$$

$$\mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \quad \text{(linear dielectrics)}$$

$$\mathbf{D} = \epsilon_0 (1 + \chi_e)\mathbf{E} \equiv \epsilon \mathbf{E} \quad \text{(linear dielectrics)}$$

$$\epsilon_r \equiv 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$$