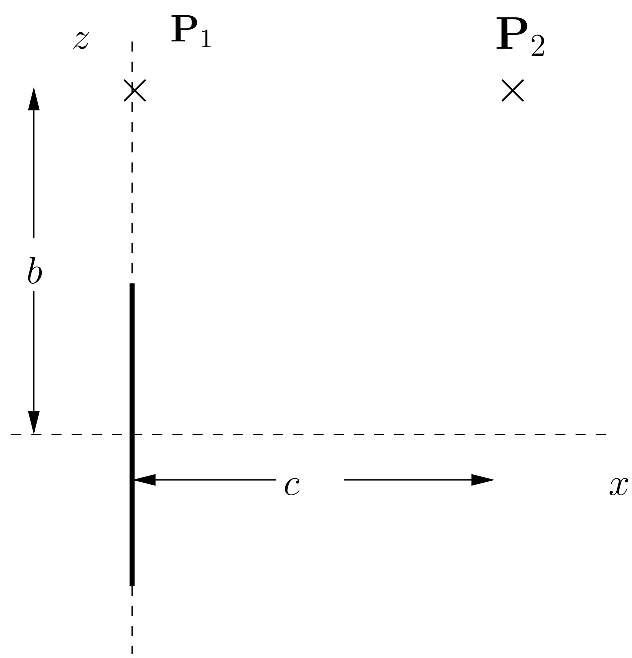
PHYS 333 — Exam #2
Friday, October 18, 2013

Name:________________________

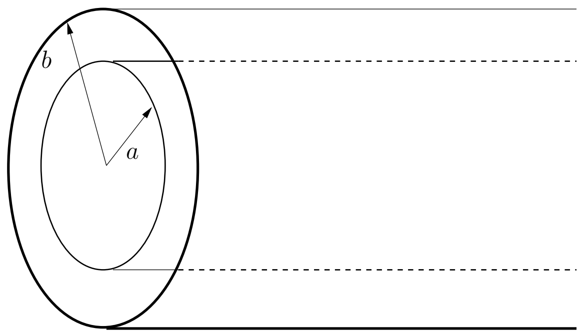
1. (20 pts) Consider a line of charge of centered on the origin with a length L and linear charge density $\lambda(z) = -kz$.



   (a) Determine an expression for the electric potential at a point P_1 a distance b above the center of the line of charge. Express your answer in the form of a definite integral that could be evaluated by a computer algebra system.

   (b) Determine an expression for the electric potential at the illustrated point P_2 a distance c to the right of point P_1 charge. Express your answer in the form of a definite integral that could be evaluated by a computer algebra system.

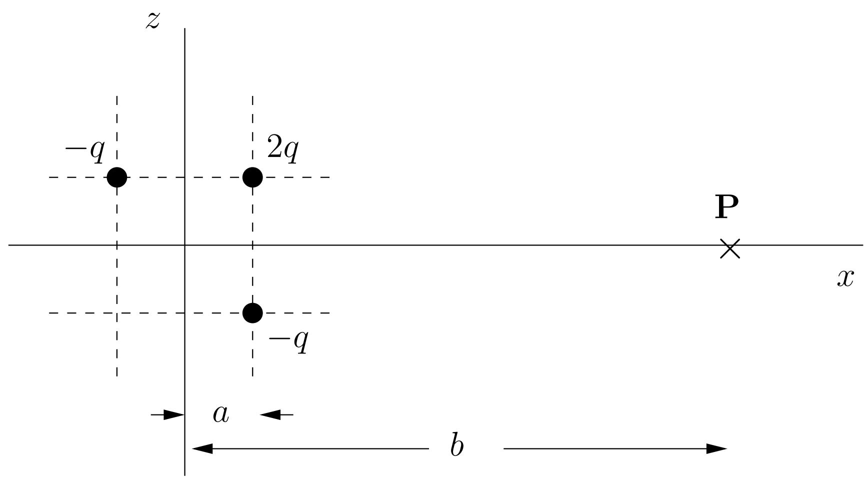
2. (20 pts) Consider two infinitely long concentric thin conducting cylinders, the inner cylinder with radius a, and the outer cylinder with radius b. The region between the two cylinders ($a < s < b$) is filled with a linear dielectric material with susceptibility χ_e (or equivalently permittivity $\epsilon = \epsilon_0 \epsilon_r$). A charge density σ is placed on the surface of the inner cylinder at $s = a$.



(a) Determine expressions for the electric field, the electric displacement, and the polarization everywhere.

(b) Determine expressions for all bound charges.

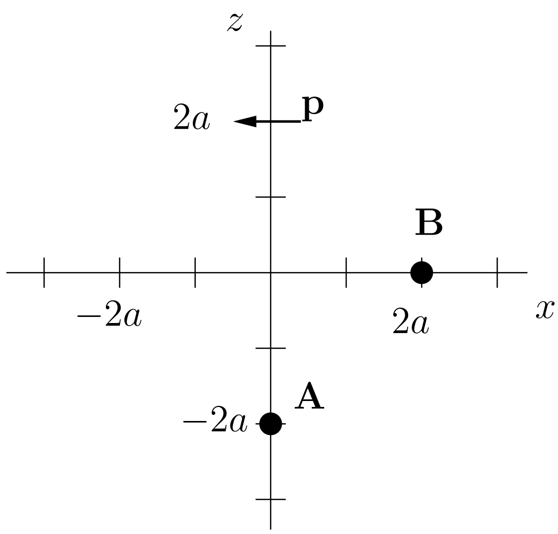
3. (20 pts) Consider the illustrated arrangement of three charges.



(a) Calculate the dipole moment of this distribution of charges.

(b) Use your dipole moment to determine an approximation for the electric potential at points that are far away "to the right" along the x-axis, i.e, valid when $b \gg a$.

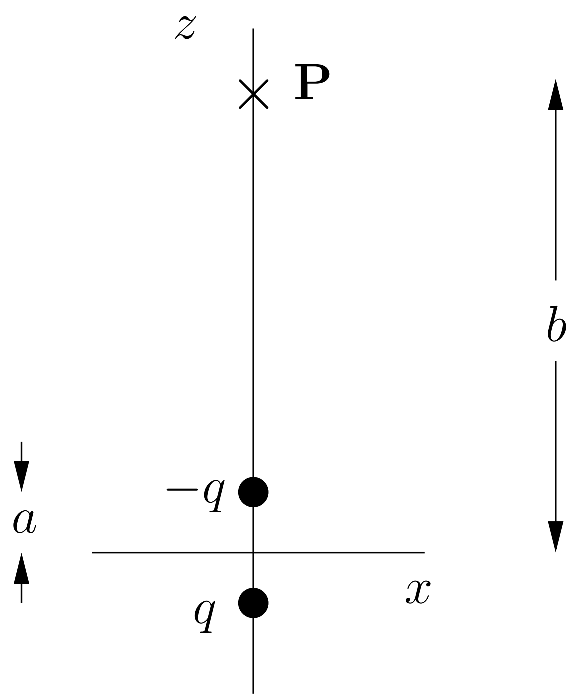
4. (20 pts) Consider an ideal dipole moment $\mathbf{p} = -p\,\hat{\mathbf{x}}$ located at the illustrated position on the z-axis a distance $2a$ above the origin.



(a) Determine the force (due to the dipole) on a point particle with a charge q located at Point **A**. Give your answer in terms of p, a, q, and Cartesian basis vectors.

(b) Determine the force (due to the dipole) on a point particle with a charge q located at Point **B**. You may leave your answer in terms of spherical basis vectors, but you should draw and label the appropriate basis vectors at Point **B**.

5. (20 pts) Consider the two illustrated charges centered on the origin and separated by a distance $2a$, and a point **P** on the z-axis that is very far away from both charges. Find an expression for the *exact* potential, and then use a binomial expansion to find the approximate potential at point P, far away from the charges. (Keep the first non-vanishing in your approximation.)



Equations

$$V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E}\cdot d\mathbf{l}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathscr{r}}\,d\tau' \longrightarrow \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\mathbf{r}')}{\mathscr{r}}\,da' \longrightarrow \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(\mathbf{r}')}{\mathscr{r}}\,dl' \longrightarrow \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{\mathscr{r}_i}$$

$$\mathbf{E} = -\vec{\nabla} V$$

$$\mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}} \qquad \& \qquad \frac{\partial V_{\text{above}}}{\partial n} - \frac{\partial V_{\text{below}}}{\partial n} = -\frac{1}{\epsilon_0}\sigma$$

$$W = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j=1,j>i}^{n}\frac{q_i q_j}{\mathscr{r}_{ij}} = \frac{1}{8\pi\epsilon_0}\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\frac{q_i q_j}{\mathscr{r}_{ij}}$$

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i V(\mathbf{r}_i) \longrightarrow \frac{1}{2}\int \rho V\,d\tau$$

$$W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2\,d\tau$$

$$C \equiv \frac{Q}{\Delta V}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

$$V(\mathbf{r}) = V_{\text{monopole}}(\mathbf{r}) + V_{\text{dipole}}(\mathbf{r}) + \cdots$$

$$V_{\text{monopole}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$V_{\text{dipole}} = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[3(\mathbf{p}\cdot\hat{\mathbf{r}})\,\hat{\mathbf{r}} - \mathbf{p}\right]$$

$$Q = \sum_{i=1}^{n} q_i \longrightarrow \int \rho(\mathbf{r}')\, d\tau'$$

$$\mathbf{p} = \sum_{i=1}^{n} q_i \mathbf{r}_i \longrightarrow \int \mathbf{r}' \rho(\mathbf{r}')\, d\tau'$$

$$\mathbf{E}_{\text{dip}}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)$$

$$\mathbf{N} = \mathbf{p} \times \mathbf{E}$$

$$\mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E}$$

$$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}}$$

$$\rho_b = -\nabla\cdot\mathbf{P}$$

$$\rho = \rho_b + \rho_f$$

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$$

$$\nabla\cdot\mathbf{D} = \rho_f \longrightarrow \oint \mathbf{D}\cdot d\mathbf{a} = Q_{f_{\text{enc}}}$$

$$\mathbf{P} = \epsilon_0\chi_e\mathbf{E} \quad \text{(linear dielectrics)}$$

$$\mathbf{D} = \epsilon_0(1+\chi_e)\mathbf{E} \equiv \epsilon\mathbf{E} \quad \text{(linear dielectrics)}$$

$$\epsilon_r \equiv 1 + \chi_e = \frac{\epsilon}{\epsilon_0}$$