Problem B

We have written the singlet state $|0\,0\rangle$ as a superposition of $\uparrow\downarrow$ and $\downarrow\uparrow$, where the arrows represent S_z eigenstates of each spin. However, there is nothing special about using S_z eigenstates. We could equally well write the singlet in terms of S_x eigenstates if we had a notation up to the task. I would suggest the following:

$$|{+z},{+z}\rangle = \uparrow\uparrow, \qquad |{+z},{-z}\rangle = \uparrow\downarrow, \qquad |{-z},{+z}\rangle = \downarrow\uparrow, \qquad |{-z},{-z}\rangle = \downarrow\downarrow$$

Start from

$$|0\,0\rangle = \frac{1}{\sqrt{2}}\Big(|{+z},{-z}\rangle - |{-z},{+z}\rangle\Big)$$

and show that

$$|0\,0\rangle = \frac{1}{\sqrt{2}}\Big(|{+x},{-x}\rangle - |{-x},{+x}\rangle\Big).$$

Hint: see p. 175 for discussion on how to relate S_z eigenstates to S_x eigenstates.

Problem C

This problem is concerned with the specific hidden variable theory we concocted in class. Namely, that when the e^- and e^+ pair are created, the internal variable $\hat{\mathbf{n}}$ inside the electron determines the spin measurements at detectors A and B according to

$$A(\hat{\mathbf{r}},\hat{\mathbf{n}}) = \operatorname{sgn}(\hat{\mathbf{r}}\cdot\hat{\mathbf{n}}) \qquad B(\hat{\mathbf{r}},\hat{\mathbf{n}}) = -\operatorname{sgn}(\hat{\mathbf{r}}\cdot\hat{\mathbf{n}})$$

where $\hat{\mathbf{r}}$ is a dummy argument representing any orientation you choose.

(a) Show that in this theory $P(\hat{\mathbf{a}},\hat{\mathbf{b}}) = -1 + \frac{2}{\pi}\theta$, where θ is the angle between $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$.

(b) Show that this result for P is consistent with Bell's inequality for the case where $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$ all lie in a plane. (It is possible to show it more generally, but that's quite a bit of work.)