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An electron is trapped in an infinite square well of width $5 \times 10^{-9}$ m. Use Heisenberg’s uncertainty to determine a lower bound on the electron’s average kinetic energy.
Spin for $\hbar$

An electron is prepared in the $|+z\rangle$ spin state.

(a) Subsequently, a measurement is made of $S_x$, the $x$-component of the spin. What are the possible values of $S_x$ that could be obtained in this measurement.

(b) After $S_x$ is measured, what is the probability of an $S_z$ measurement giving $+\hbar/2$?
A particle of mass $m$ has the wavefunction $\psi = A \sin(kx)$ for a region of $x$ where the potential energy is uniform $U(x) = U_0$.

Determine the energy $E$ of this particle in terms of the given parameters and fundamental constants.
What is the longest wavelength of light that can be emitted from a hydrogen atom that is in the $n = 4$ state?
In Heisenberg’s Uncertainty Relation, $\Delta x \Delta p \geq \hbar/2$, we can trade between uncertainty in position and uncertainty in momentum. With this in mind:

**(a)** Draw a wavefunction for a particle having a *certain* position.

**(b)** Draw a wavefunction for a particle having a *certain* momentum. *Hint*: think about de Broglie.

**(c)** Draw a wavefunction for a particle with some uncertainty in both the position and momentum.
Spin for $2\hbar$

A spin-1/2 particle is in the state

$$|\psi\rangle = \frac{i}{2} |+z\rangle + \frac{\sqrt{3}}{2} |-z\rangle$$

Determine the probability that an $S_y$ measurement will result in the value $-\hbar/2$. 

Return
The wavefunction shown is a solution to Schrödinger’s equation for some piecewise-constant potential $U(x)$. Sketch this potential, along with the value for the energy $E$. 

\[
\begin{align*}
\psi & \quad x \\
0 & \quad 2 & \quad 4 & \quad 6 & \quad 8 & \quad 10 \\
\end{align*}
\]
Consider an particle in an infinite square well potential, where the particle is in the $n = 2$ energy level. Determine an integral expression for the probability of finding the particle in the middle third of the well.
Heisenberg for $3\hbar$

Surprise! This one is actually about Bell, not Heisenberg.

An electron-positron pair is created in the state

$$\psi = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

The electron’s $z$-component of spin is measured to be $S^\text{elec}_z = +\hbar/2$. The positron is sent through a Stern-Gerlach detector oriented at an angle $30^\circ$ with respect to the $z$-axis.

(a) What is the quantum mechanics prediction for the probability of $S^{\text{pos}}_{30^\circ} = +\hbar/2$?

(b) What is the hidden variable prediction for the probability of $S^{\text{pos}}_{30^\circ} = +\hbar/2$?
An electron-positron pair is in the state

\[ |\psi\rangle = 0.4 |↑↑\rangle + 0.2 |↑↓\rangle + 0.8 |↓↑\rangle + 0.4 |↓↓\rangle \]

(a) Rewrite this state as

\[ |\psi\rangle = c_+ |↑\rangle |\phi_1\rangle + c_- |↓\rangle |\phi_2\rangle \]

and determine \( c_+ \), \( c_- \), \( |\phi_1\rangle \) and \( |\phi_2\rangle \).

(b) Is this state entangled or separable?
Schrödinger for $3\hbar$

A particle of mass $m$ in an infinite square well of width $L$ is initially in the superposition

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}} \psi_1(x) + \frac{1}{\sqrt{2}} \psi_2(x)$$

where $\psi_1(x)$ is the wavefunction corresponding to energy $E_1$ and $\psi_2(x)$ is the wavefunction corresponding to energy $E_2$.

(a) Write down the time-dependent wavefunction $\Psi(x, t)$.

(b) The resulting probability density $P(x, t)$ oscillates with some period $T$. Find an expression for this period in terms of the given parameters.
Consider a two particles, each of which can be in one of four possible states: $|A\rangle$, $|B\rangle$, $|C\rangle$, and $|D\rangle$. Each state is equally likely to occur.

(a) What is the probability of the two particles being in the same state if they are distinguishable particles?

(b) What is the probability of the two particles being in the same state if they are identical bosons?

(c) How many different states are possible if the two particles are identical fermions?