Problem F

The goal is to evaluate the integral

\[ \int_{0}^{\pi/2} \tan x \delta \left( \sin x - \frac{1}{2} \right) \, dx \]

(a) Evaluate the integral via the variable substitution \( u = \sin x \).

(b) This time, evaluate the integral by first using the \( \delta \)-function relation

\[ \delta(f(x)) = \sum \delta(x - x_i) \left. \frac{df}{dx} \right|_{x=x_i} \]

where \( f(x_i) = 0 \).

Note: you only need to keep terms in the sum on \( i \) that lie in the range of integration.

Problem G

Consider a variation on the neutral current presented in Section 12.3.1. As before, in frame \( S \) the positive charges and negative charges \( (\pm q_0) \) are equally spaced with separation \( \ell_0 \), so the charge densities are

\[ \lambda_{\pm} = \pm \lambda_0 = \pm q_0/\ell_0 \]

However, consider the case where the positive charges are stationary in \( S \), while the negative charges are moving at speed \( v \). Also, take the charge \( q \) to be moving with the same speed \( v \), in the same direction as the negative charges.

(a) Determine the current in frame \( S \), and from the current, determine the force on charge \( q \).

(b) Now consider a frame \( \bar{S} \) in which the charge \( q \) is at rest, as well as the negative charges \( (-q_0) \) in the wire. Determine the new charge densities \( \lambda_{\bar{+}} \) and \( \lambda_{\bar{-}} \) in \( \bar{S} \).

(c) Using your answer from (b), determine the force on the charge \( q \).

(d) Comment on the relationship between your answers in parts (a) and (c).