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Corrigendum

Corrigendum: Anomalous dimension in a two-species reaction–diffusion system (2018 *J. Phys. A: Math. Theor.* 51 034002)

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We make two corrections to the renormalization group calculation presented in Vollmayr-Lee *et al* [1]. First, the field renormalization technique presented is not applicable for the B particle density in $d = 2$ because of noncommutivity of the $\epsilon \to 0$ and $t \to \infty$ limits. The B particle density in $d < 2$ and the correlation function for $d \leqslant 2$ are unaffected by this issue. Second, we correct a symmetry factor in one of the diagrams, which modifies the correlation function scaling exponents.

The B particle density at the upper critical dimension $d = 2$ decays as $\langle b \rangle \sim t^{-\theta}(\ln t)^{\alpha}$. Our calculation of α was based on the assumption that the contribution $A(z)/\epsilon^2$ in equation (25) was asymptotically negligible because

$$A(z) \propto (z - z^*) \sim \begin{cases} t^{-\epsilon/2} & d < 2 \\ 1/\ln t & d = 2. \end{cases} \tag{1}$$

This assumption is valid for $d < 2$ since for any $\epsilon > 0$, there exists an asymptotic regime where $1/(\epsilon^2 t^{\epsilon/2})$ is negligibly small. But for $d = 2$ we must take the $\epsilon \to 0$ limit before the large t limit, and this term cannot be neglected. As a result, the field renormalization technique employed in [1] is not applicable and one must instead employ the technique of Rajesh and Zaboronski [2], where they renormalize instead the logarithmic derivative $t\partial_t \ln\langle b \rangle$. Their result agrees with our [1] equation (7), but their value of α, which in our notation reads

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$$\alpha = \left(\frac{1+\delta}{2-p}\right)\left[\frac{3}{2} + \ln\left(\frac{1+\delta}{2}\right) + \frac{1}{2}\left(\frac{1+\delta}{2-p}\right) f(\delta)\right] - \frac{4\pi(1+\delta)}{2-p}\left(\frac{1}{\lambda} - \frac{1+\delta}{\lambda'}\right), \tag{2}$$

corrects the value we reported in equation (8).

However, the problem of the noncommuting $\epsilon \to 0$ and $t \to \infty$ limits does not affect the calculation of correlation function scaling exponents ϕ and α_2, defined via

$$\tilde{C}_{BB}(r,t) = \begin{cases} t^{\phi} f(r/\sqrt{t}) & d < 2 \\ (\ln t)^{\alpha_2} f(r/\sqrt{t}) & d = 2, \end{cases} \tag{3}$$

provided one renormalizes the *scaled* correlation function $\tilde{C}_{BB}(r,t) = C_{BB}(r,t)/\langle b(t)\rangle^2$. Our bare expansion for the unscaled C^B_{BB} in equation (40) is then replaced by

$$\hat{\tilde{C}}^B_{BB} = \frac{t\lambda'^2 h(Q)}{\lambda}\left[1 + \lambda t^{\epsilon/2}\left(\frac{C(z) - 2A(z)}{\epsilon^2} + \frac{D(z) - 2B(z)}{\epsilon} + \ldots\right)\right]. \tag{4}$$

The $1/\epsilon^2$ term then vanishes because $C(z) = 2A(z)$. The remaining $1/\epsilon$ term can be controlled by field renormalization as before, with the same final results:

$$\phi = \left[\pi\Big(D(z^*) - 2B(z*)\Big) + \frac{1}{2}\right]\epsilon + O(\epsilon^2) \tag{5}$$

for $d < 2$ and

$$\alpha_2 = 2\pi\Big(D(z^*) - 2B(z^*)\Big) \tag{6}$$

for $d = 2$.

The second error in [1] was a symmetry factor of two in the first diagram of Class 2 in figure A2. The corrected equation (A.3) reads

$$F_2 = 12Qz(8\pi)^{\epsilon/2}\left(\frac{z}{z^*} - 1\right)\frac{1}{\epsilon^2} + \left(6 + 24Qz - 8Q + 3Q^2z^2 f(\delta) - \frac{15Qz^2}{1+\delta}\right)\frac{1}{\epsilon} \tag{7}$$

which changes $D(z^*)$ in equation (41) to

$$D(z^*) = -\frac{9 - 13Q}{6\pi(3-2Q)} + \frac{3Q(1+\delta)}{2\pi} + \frac{Q^2(1+\delta)^2 f(\delta)}{2\pi} + O(\epsilon). \tag{8}$$

This modifies the correlation function exponents: equation (6) becomes

$$\phi = \frac{7}{24 - 18p}\epsilon + O(\epsilon^2) \tag{9}$$

and equation (10) becomes

$$\alpha_2 = -\frac{5-9p}{12-9p}. \tag{10}$$

In the text after equation (11), the value of ϕ for the truncated RG expansion in $d = 1$ with $p = 1$ and $\delta = 1$ is $\phi = \frac{7}{6} \simeq 1.17$.

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References

[1] Vollmayr-Lee B, Hanson J, McIsaac R S and Hellerick J D 2018 *J. Phys. A: Math. Theor.* **51** 034002
[2] Rajesh R and Zaboronski O 2004 *Phys. Rev.* E **70** 036111