Div, Grad, and Curl	Coulomb and Gauss	Potential and Work	Miscellaneous
q	q	q	q
2q	2q	2q	2q
3q	3q	3q	3q

Given a general scalar field $T(\mathbf{r})$ and a vector field $\mathbf{v}(\mathbf{r})$, which second derivatives can be taken that are identically zero?

Answer: $\nabla \times \nabla T = 0$ and $\nabla \cdot (\nabla \times \mathbf{v}) = 0$.

◂ Return

For the vector field

$$\mathbf{v} = -xy^2\,\hat{\mathbf{x}} + zy\,\hat{\mathbf{y}} - 6\,\hat{\mathbf{z}}$$

determine the line integral $\int_{\mathcal{P}} \mathbf{v} \cdot d\mathbf{l}$ from the point $(0,0,0)$ to $(1,1,0)$ along the path $y = x^2$, $z = 0$.

Answer: $-1/6$

◂ Return

For the vector field (in cylindrical coordinates)

$$\mathbf{v} = s(1 + \cos\phi)\,\hat{\mathbf{s}} + s(1 + \sin\phi)\,\hat{\boldsymbol{\phi}} - sz\,\hat{\mathbf{z}}$$

determine the surface integral $\int_{\mathcal{S}} \mathbf{v} \cdot d\mathbf{a}$ over the cylindrical surface defined by $s = R$ and $0 < z < L$.

Answer: $2\pi L R^2$

◂ Return

Given the electric field

$$\mathbf{E}(\mathbf{r}) = 2xy\,\hat{\mathbf{x}} + (x^2 - 2yz)\,\hat{\mathbf{y}} - y^2\,\hat{\mathbf{z}}$$

determine the charge density $\rho(\mathbf{r})$.

Answer: $\epsilon_0(2y - 2z)$

◂ Return

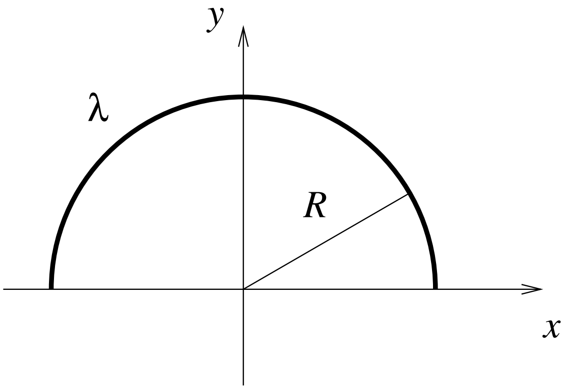
Given the charge density $\rho(\mathbf{r}) = kr$, determine the electric field $\mathbf{E}(\mathbf{r})$.

Answer:

$$\mathbf{E} = \frac{\pi k R^4}{4\epsilon_0}\hat{\mathbf{r}}$$

◂ Return

Given a half-circle of charge density λ as shown, determine $\mathbf{E}$ at the origin.



Answer:

$$\mathbf{E} = -\frac{\lambda}{2\pi\epsilon_0 R}\hat{\mathbf{y}}$$

Return

$$\frac{1}{2}\int_{\mathcal{V}} \rho V \, d\tau = \frac{\epsilon_0}{2}\int_{\mathcal{V}} E^2 \, d\tau$$

Is this a true statement? Why or why not?

Answer: No. The E^2 integral must run over all space. Otherwise there is an additional boundary term from the integration by parts.

Given a sphere of radius R with uniform surface charge density σ, determine the potential V at the point $\mathbf{r} = (R/2)\,\hat{\mathbf{z}}$.

Answer: Same as any point on or inside sphere, since $\mathbf{E} = 0$. $V = R\sigma/\epsilon_0$.

Return

Calculate the energy store in the following charge configuration: a sphere of radius a with total charge q spread uniformly on the surface, surrounded by a concentric sphere of radius $b > a$ with total charge $-q$ also uniformly spread on the surface.

Answer:

$$W = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

Return

Evaluate this integral:

$$\int r^2 \sin(\mathbf{r} \cdot \mathbf{a}) \delta^3(\mathbf{b} - \mathbf{r}) \, d\tau$$

where integration runs over all space.

Answer: $b^2 \sin(\mathbf{b} \cdot \mathbf{a})$

◂ Return

Given the potential $V(\mathbf{r}) = xy^2z^3$, determine the charge density $\rho(\mathbf{r})$.

Answer: $\rho = -(2xz^3 + 6xy^2z)/\epsilon_0$.

◂ Return

A conducting spherical shell of inner radius a and outer radius b carries a total charge Q. A point charge q is placed at the center. Determine the surface charge densities on the inner and outer surfaces of the conductor. **Answer:**

$$\sigma_a = -\frac{q}{4\pi a^2} \quad \sigma_b = \frac{Q+q}{4\pi b^2}$$

Return