PHYS 333 Test 3 Review

December 10, 2006

Linear Dielectric

Consider infinite parallel plates, one at $z = 0$ with charge density σ and the other at $z = d$ with charge density $-\sigma$.

In between the plates is a linear dielectric (so $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$) whose susceptibility varies with distance as $\chi_e = 2z$.

Find $\mathbf{D}$, $\mathbf{E}$, $\mathbf{P}$, and the bound charge.

Answer: $\mathbf{D} = \sigma\,\hat{\mathbf{z}}$, $\mathbf{E} = \frac{\sigma}{\epsilon_0(1+2z)}\,\hat{\mathbf{z}}$, $\mathbf{P} = \frac{2\sigma z}{1+2z}\,\hat{\mathbf{z}}$
$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}} = 0$ at bottom
$\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}} = \frac{2\sigma d}{1+2d}$ at top
$\rho_b = -\nabla\cdot\mathbf{P} = 2\sigma\frac{2z-1}{(2z+1)^2}$

Magnetic Forces and Biot-Savart

On the board is shown a segment of a current loop, and a positively charged particle with a particular velocity.

Determine the direction of the force on the particle, or if the force is zero.

Ampere's Law

A current flows down a cylindrical tube of radius a oriented along the z-axis. The current density is given by $\mathbf{J} = ks\,\hat{\mathbf{z}}$.

Determine $\mathbf{B}$.

Answer: $\mathbf{B} = \frac{\mu_0 k s^2}{3}\hat{\phi}$ for $s < a$
$\mathbf{B} = \frac{\mu_0 k a^3}{3s}\hat{\phi}$ for $s > a$

Vector Potential

Same setup as before:

A current flows down a cylindrical tube of radius a oriented along the z-axis. The current density is given by $\mathbf{J} = ks\,\hat{\mathbf{z}}$.

Determine $\mathbf{A}$.

Answer: $\mathbf{A} = -\frac{\mu_0 k s^3}{9}\hat{\mathbf{z}}$ for $s < a$ where I've chosen to put the zero of $\mathbf{A}$ at $s = 0$.
$\mathbf{A} = -\frac{\mu_0 k a^3}{3}\ln(s/a) - \frac{\mu_0 k a^3}{9}\hat{\mathbf{z}}$ for $s > a$

Boundary Conditions

Derive the magnetic field just inside a circular solenoid (current I, n turns per length) by treating the current as a surface current $\mathbf{K}$ in the $\hat{\phi}$ direction.

Answer: see Example 5.9, p. 227

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Can you give a one-line argument why this field inside must be uniform?

No, but here's two sentences:
Looking at Fig. 5.37: the vertical part of loop 2 that is inside the cylinder provides the only contribution to $\oint \mathbf{B} \cdot d\mathbf{l}$, so the current enclosed determines $\mathbf{B}$ at the location of this loop segment. But the current enclosed doesn't depend on the location of this segment, so $\mathbf{B}$ must be uniform inside solenoid.

Magnetic Dipoles and Magnetization

A long circular cylinder of radius a has a magnetization $\mathbf{M} = ks\,\hat{\phi}$. Determine the bound currents and magnetic field inside and outside the cylinder.

Answer: $\mathbf{J}_b = 2k\,\hat{\mathbf{z}}$ for $s < a$, and $\mathbf{J}_b = 0$ for $s > a$.
$\mathbf{K}_b = -ka\,\hat{\mathbf{z}}$ at $s = a$.
$\mathbf{B} = \mu_0 ks\,\hat{\phi}$ for $s < 0$ and $\mathbf{B} = 0$ for $s > a$.