Consider infinite parallel plates, one at \( z = 0 \) with charge density \( \sigma \) and the other at \( z = d \) with charge density \( -\sigma \).

In between the plates is a linear dielectric (so \( \mathbf{P} = \epsilon_0 \chi_e \mathbf{E} \)) whose susceptibility varies with distance as \( \chi_e = 2z \).

Find \( \mathbf{D} \), \( \mathbf{E} \), \( \mathbf{P} \), and the bound charge.

**Answer:** \( \mathbf{D} = \sigma \hat{z} \), \( \mathbf{E} = \frac{\sigma}{\epsilon_0 (1+2z)} \hat{z} \), \( \mathbf{P} = \frac{2\sigma z}{1+2z} \hat{z} \)

\( \sigma_b = \mathbf{P} \cdot \hat{n} = 0 \) at bottom

\( \sigma_b = \mathbf{P} \cdot \hat{n} = \frac{2\sigma d}{1+2d} \) at top

\( \rho_b = -\nabla \cdot \mathbf{P} = 2\sigma \frac{2z-1}{(2z+1)^2} \)
On the board is shown a segment of a current loop, and a positively charged particle with a particular velocity.

Determine the direction of the force on the particle, or if the force is zero.
A current flows down a cylindrical tube of radius $a$ oriented along the $z$-axis. The current density is given by $\mathbf{J} = ks \mathbf{\hat{z}}$.

Determine $\mathbf{B}$.

**Answer:**

$\mathbf{B} = \frac{\mu_0 ks^2}{3} \phi$ for $s < a$

$\mathbf{B} = \frac{\mu_0 ka^3}{3s} \phi$ for $s > a$
Same setup as before:

A current flows down a cylindrical tube of radius $a$ oriented along the $z$-axis. The current density is given by $\mathbf{J} = ks\hat{z}$.

Determine $\mathbf{A}$.

**Answer:** $\mathbf{A} = -\frac{\mu_0 k s^3}{9}\hat{z}$ for $s < a$ where I’ve chosen to put the zero of $\mathbf{A}$ at $s = 0$.

$\mathbf{A} = -\frac{\mu_0 k a^3}{3} \ln(s/a) - \frac{\mu_0 k a^3}{9}\hat{z}$ for $s > a$
Derive the magnetic field just inside a circular solenoid (current $I$, $n$ turns per length) by treating the current as a surface current $\mathbf{K}$ in the $\hat{\phi}$ direction.

**Answer:** see Example 5.9, p. 227
Derive the magnetic field just inside a circular solenoid (current $I$, $n$ turns per length) by treating the current as a surface current $\mathbf{K}$ in the $\hat{\phi}$ direction.

**Answer:** see Example 5.9, p. 227

Can you give a one-line argument why this field inside must be uniform?

No, but here’s two sentences: Looking at Fig. 5.37: the vertical part of loop 2 that is inside the cylinder provides the only contribution to $\oint \mathbf{B} \cdot d\mathbf{l}$, so the current enclosed determines $\mathbf{B}$ at the location of this loop segment. But the current enclosed doesn’t depend on the location of this segment, so $\mathbf{B}$ must be uniform inside solenoid.
A long circular cylinder of radius $a$ has a magnetization $\mathbf{M} = ks \hat{\phi}$. Determine the bound currents and magnetic field inside and outside the cylinder.

**Answer:** $\mathbf{J}_b = 2k \hat{z}$ for $s < a$, and $\mathbf{J}_b = 0$ for $s > a$.

$\mathbf{K}_b = -ka \hat{z}$ at $s = a$.

$\mathbf{B} = \mu_0 ks \hat{\phi}$ for $s < 0$ and $\mathbf{B} = 0$ for $s > a$. 