Review Questions for Chapter 7

1. A wire loop encloses a solenoid, as shown. The solenoid carries a varying current \( I(t) = I_0 \cos(\omega t) \). Determine the current in the outer loop.

\[ I_{\text{outer}} = \frac{\mu_0 n I_0 \omega \pi a^2}{R} \sin(\omega t) \]

2. Consider two "loops": one is a straight line wire (that loops back around to itself somewhere infinitely far away) and the other is a rectangular loop. The loops lie in the same plane. Determine the mutual inductance \( M \).

\[ M = \frac{\mu_0 h}{2\pi} \ln \left( \frac{b}{a} \right) \]

3. Show that Ampere’s Law is inconsistent with charge conservation.

**Answer:** Take divergence: \( \nabla \cdot (\nabla \times \mathbf{B}) = \mu_0 \nabla \cdot \mathbf{J} \). The left side is zero for any vector field \( \mathbf{B} \). The right side should equal \(-\mu_0(\partial \rho / \partial t)\) by the continuity equation, so these are inconsistent.
4. A magnetic rail gun is a U-shaped loop of wire, with a sliding bar across it that completes a circuit. The goal is to get the bar accelerating to the right. Find two different ways to make this happen (using $B$ fields and currents).

Answer: (a) standard method: put a battery in the loop so that current flows clockwise, and place loop in a magnetic field pointing into page. (b) alternate method: no battery; just start with a large $B$ field pointing into the page, and then reduce its strength. This will accelerate the loop to the right until $B = 0$. (What happens next?).