Problem C

The goal is to evaluate the integral

$$\int_0^{\pi/2} \tan x \, \delta\big(\sin x - \tfrac{1}{2}\big) \, dx$$

(a) Evaluate the integral via the variable substitution $u = \sin x$.

(b) This time, evaluate the integral by first using the δ-function relation

$$\delta\Big(f(x)\Big) = \sum_i \frac{\delta(x - x_i)}{\left|\left(\dfrac{df}{dx}\right)_{x=x_i}\right|} \quad \text{where} \quad f(x_i) = 0.$$

Note: you only need to keep terms in the sum on i that lie in the range of integration.