Problem F

The goal is to evaluate the integral

$$\int_0^{\pi/2} \tan x \, \delta\big(\sin x - \tfrac{1}{2}\big) \, dx$$

(a) Evaluate the integral via the variable substitution $u = \sin x$.

(b) This time, evaluate the integral by first using the δ-function relation

$$\delta\Big(f(x)\Big) = \sum_i \frac{\delta(x - x_i)}{\left|\left(\dfrac{df}{dx}\right)_{x=x_i}\right|} \quad \text{where} \quad f(x_i) = 0.$$

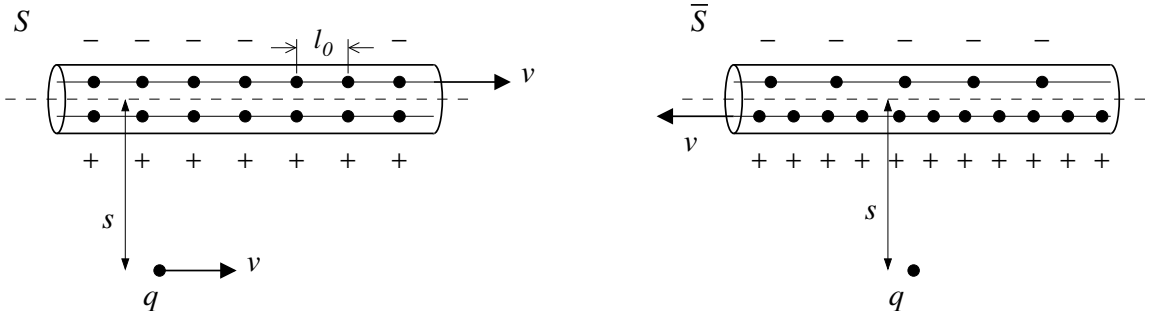
Note: you only need to keep terms in the sum on i that lie in the range of integration.

Problem G

Consider a variation on the neutral current presented in Section 12.3.1. As before, in frame S the positive charges and negative charges ($\pm q_0$) are equally spaced with separation ℓ_0, so the charge densities are

$$\lambda_\pm = \pm\lambda_0 = \pm q_0/\ell_0$$

However, consider the case where the positive charges are stationary in S, while the negative charges are moving at speed v. Also, take the charge q to be moving with the same speed v, in the same direction as the negative charges.



(a) Determine the current in frame S, and from the current, determine the force on charge q.

(b) Now consider a frame $\bar{S}$ in which the charge q is at rest, as well as the negative charges ($-q_0$) in the wire. Determine the new charge densities $\bar{\lambda}_+$ and $\bar{\lambda}_-$ in $\bar{S}$.

(c) Using your answer from (b), determine the force on the charge q.

(d) Comment on the relationship between your answers in parts (a) and (c).