PHYS 334 Electromagnetic Theory II

In Class Exercise 1 — January 17, 2024 Name: Solutions

1. Use the divergence theorem, Eq. (1.56), to convert Gauss's law from differential to integral form. Do these two forms have identical content, i.e., does one imply the other? Argue why or why not.

Divergence theorem: $\int_V (\vec{\nabla}\cdot\vec{v})\,d\tau = \oint_S \vec{v}\cdot d\vec{a}$

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \Rightarrow \int_V (\vec{\nabla}\cdot\vec{E})\,d\tau = \frac{1}{\varepsilon_0}\int_V \rho\,d\tau$$

LHS: $\int_V (\vec{\nabla}\cdot\vec{E})\,d\tau = \oint_S \vec{E}\cdot d\vec{a}$

RHS: $\frac{1}{\varepsilon_0}\int_V \rho\,d\tau = Q_{enc}/\varepsilon_0$

So $\boxed{\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}}$ ✓

They have identical content. Starting from integral form we can show $\int_V \vec{\nabla}\cdot\vec{E}\,d\tau = \frac{1}{\varepsilon_0}\int_V \rho\,d\tau$ since this must hold for any volume V, the integrands must be equal

2. Use Stokes' Theorem, Eq. (1.57), to convert Faraday's law from differential to integral form. Do these two forms have identical content, i.e., does on imply the other? Argue why or why not.

Stokes' theorem: $\int_S (\vec{\nabla}\times\vec{v})\cdot d\vec{a} = \oint_P \vec{v}\cdot d\vec{l}$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \Rightarrow \int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{a} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$$

LHS: $\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{a} = \oint_P \vec{E}\cdot d\vec{l}$

RHS: $-\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{a} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{a} = -\frac{d\Phi_B}{dt}$

So $\boxed{\oint_P \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$ ✓

Again identical content. From the integral form we can show $\int_S (\vec{\nabla}\times\vec{E})\cdot d\vec{a} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{a}$ which holds for any surface S, so the integrands must be equal.