PHYS 334 Electromagnetic Theory II

In Class Exercise 3 — January 22, 2024 **Name:** Solutions

1. Given the definition $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, show that Gauss's law can be written as $\nabla \cdot \mathbf{D} = \rho_f$.

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 = \frac{\rho_f}{\varepsilon_0} + \frac{\rho_b}{\varepsilon_0}$$

$\rho_b = -\vec{\nabla}\cdot\vec{P}$, so $\vec{\nabla}\cdot\vec{E} = \frac{\rho_f}{\varepsilon_0} - \vec{\nabla}\cdot\left(\frac{\vec{P}}{\varepsilon_0}\right)$

multiply through by ε_0 to get

$$\vec{\nabla}\cdot(\varepsilon_0\vec{E}) + \vec{\nabla}\cdot\vec{P} = \rho_f$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f \checkmark$$

2. A point charge q is placed in a linear dielectric with dielectric constant ϵ_r.

(a) Determine $\mathbf{D}$.

From $\vec{\nabla}\cdot\vec{D} = \rho_f \Rightarrow \oint \vec{D}\cdot d\vec{a} = q_{enc,f}$

we find $\boxed{\vec{D} = \frac{q}{4\pi r^2}\hat{r}}$

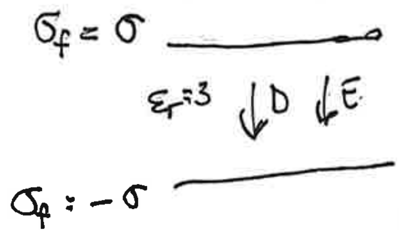
(b) Use your answer from part (a) to determine $\mathbf{E}$.

Since it's linear, $\vec{D} = \varepsilon_0\varepsilon_r\vec{E}$ or $\vec{D} = \varepsilon\vec{E}$, so

$$\boxed{\vec{E} = \frac{q}{4\pi\varepsilon r^2}\hat{r}}$$

3. Consider a capacitor made of parallel plates with free charge density σ and $-\sigma$, with a linear dielectric material in the middle with dielectric constant $\epsilon_r = 3$.

(a) Find **D** in the interior of the capacitor.



From Gauss's Law $\boxed{D = \sigma \downarrow}$

(b) Find **E** in the interior of the capacitor.

$$\vec{E} = \frac{\vec{D}}{\varepsilon} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r} \Rightarrow \boxed{E = \frac{\sigma}{3\varepsilon_0} \downarrow}$$

(c) Find **P** in the interior of the capacitor.

$$\vec{P} = \vec{D} - \varepsilon_0 \vec{E} \Rightarrow P = \sigma - \frac{\sigma}{3} = \boxed{\frac{2\sigma}{3} \downarrow}$$

(d) Determine the bound charge on each plate of the capacitor.

$\sigma_b = \vec{P} \cdot \hat{n}$ where $\hat{n}$ points outward from material, so $\hat{n}$ is up at the top and down at the bottom

top: $\sigma_b = \frac{2\sigma}{3}(-\hat{j}) \cdot \hat{j} = -\frac{2\sigma}{3}$

bottom: $\sigma_b = \frac{2\sigma}{3}(-\hat{j}) \cdot (-\hat{j}) = +\frac{2\sigma}{3}$