PHYS 334 Electromagnetic Theory II

In Class Exercise 10 — February 12, 2024 **Name:** Solutions

1. Show that the sinusoidal waves $f(z,t) = A\cos[kz - \omega t + \delta]$ and $g(z,t) = A\cos[-kz - \omega t + \delta]$ satisfy the wave equation $\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$. Find v in terms of ω and k.

$$\frac{\partial f}{\partial z} = -A(\sin[\ \ ])k \qquad \frac{\partial^2 f}{\partial z^2} = -A(\cos[\ \ ])k^2$$

$$\frac{\partial f}{\partial t} = -A(\sin[\ \ ])(-\omega) \qquad \frac{\partial^2 f}{\partial t^2} = -A(\cos[\ \ ])(-\omega)^2$$

$$\Rightarrow \frac{\partial^2 f}{\partial z^2} = \frac{k^2}{\omega^2}\frac{\partial^2 f}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \quad \checkmark$$

For g the only difference is that the chain rule for the z derivatives gives a $-k$ instead of a k. But $(-k)^2 = k^2$, so same result.

2. With separation of variables, the wave equation becomes two equations: $\frac{d^2 Z}{dz^2} = -k^2 Z$ and $\frac{d^2 T}{dt^2} = -k^2 v^2 T$. Solve these for $Z(z)$ and $T(t)$ and then multiply to get $\tilde{f}(z,t) = Z(z)T(t)$.

mistake in problem statement

- $$\frac{d^2 Z(z)}{dz^2} = -k^2 Z(z)$$

Solution is either sin & cos, or $e^{\pm ikz}$. We'll take the latter

$$Z(z) = Ae^{ikz} + Be^{-ikz}$$

- $$\frac{d^2 T}{dt^2} = -k^2 v^2 T \Rightarrow T(t) = Ce^{ikvt} + De^{-ikvt}$$

$$\Rightarrow \tilde{f}(z,t) = ACe^{ik(z+vt)} + BDe^{-ik(z+vt)} + ADe^{ik(z-vt)} + BCe^{-ik(z-vt)}$$