PHYS 334 Electromagnetic Theory II

In Class Exercise 11 — February 14, 2024 **Name:** Solutions

1. Given the results

$$\tilde{A}_R = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)\tilde{A}_I, \qquad \tilde{A}_T = \left(\frac{2v_2}{v_2 + v_1}\right)\tilde{A}_I,$$

show that the transmitted wave must be exactly in phase with the incident wave, and that the reflected wave is exactly in phase with the incident wave if $v_2 > v_1$, but is out of phase by π (or 180°) if $v_2 < v_1$.

$$\tilde{A}_R = A_R e^{i\delta_R} \qquad \tilde{A}_I = A_I e^{i\delta_I} \qquad \tilde{A}_T = A_T e^{i\delta_T}$$

so

$$A_R e^{i\delta_R} = \frac{v_2 - v_1}{v_2 + v_1} A_I e^{i\delta_I} \Rightarrow \text{either } \delta_R = \delta_I \text{ (if } v_2 > v_1)$$

$$\text{or } \delta_R = \delta_I + \pi \text{ (if } v_2 < v_1)$$

2. Given the plane wave $\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}$, show that $\nabla \cdot \tilde{\mathbf{E}} = 0$ implies the z-component of $\tilde{\mathbf{E}}$ is zero, i.e., the wave is transverse.

$$\vec{\nabla} \cdot \vec{\tilde{E}}_0 e^{i(kz-\omega t)} = \hat{z} \cdot \frac{\partial}{\partial t} \vec{\tilde{E}} e^{i(kz-\omega t)}$$

$$= \hat{z} \cdot ik \vec{\tilde{E}} e^{i(kz-\omega t)} = 0$$

requires $\hat{z} \cdot \vec{\tilde{E}} = 0$

3. Given the plane wave $\tilde{\mathbf{E}}(z,t) = \tilde{\mathbf{E}}_0 e^{i(kz-\omega t)}$ evaluate $\nabla \times \tilde{\mathbf{E}}$

$$\vec{\nabla} \times \vec{\tilde{E}}_0 e^{i(kz-\omega t)} = ?$$

Since $\vec{\tilde{E}}$ only depends on z, only z derivatives survive the curl:

$$\vec{\nabla} \times \vec{\tilde{E}}_0 e^{i(kz-\omega t)} = -\frac{\partial}{\partial z} \tilde{E}_{0,y} e^{i(kz-\omega t)} \hat{x} + \frac{\partial}{\partial z} \tilde{E}_{0,x} e^{i(kz-\omega t)} \hat{y}$$

$$= -ik \tilde{E}_{0,y} \hat{x} + ik \tilde{E}_{0,x} \hat{y}$$