PHYS 334 Electromagnetic Theory II

In Class Exercise 13 — February 19, 2024 **Name:** Solutions

1. For a normally incident plane wave with $\tilde{\mathbf{E}}_I(z,t) = \tilde{E}_{0,I} e^{i(kz-\omega t)}\,\hat{\mathbf{x}}$ and $\tilde{\mathbf{B}}_I(z,t) = \frac{\tilde{E}_{0,I}}{v_1} e^{i(kz-\omega t)}\,\hat{\mathbf{y}}$, write out the corresponding E and B fields for the reflected and transmitted waves.

$$\vec{\tilde{E}}_R = \tilde{E}_{0,R}\, e^{i(-k_1 z-\omega t)}\,\hat{x} \qquad \vec{\tilde{E}}_T = \tilde{E}_{0,T}\, e^{i(k_2 z-\omega t)}\,\hat{x}$$

(same $\vec{\tilde{E}}$ direction, opposite propagation direction for reflected wave.)

$$\vec{\tilde{B}}_R = \frac{\tilde{E}_{0,R}}{v_1} e^{i(-k_1 z-\omega t)}(-\hat{y}) \qquad \vec{\tilde{B}}_T = \frac{\tilde{E}_{0,T}}{v_2} e^{i(k_2 z-\omega t)}\,\hat{y}$$

using $\tilde{B}_0 = \frac{\tilde{E}_0}{v}$ and $\vec{E}\times\vec{B} \propto$ prop. direction

2. Using $v_1 = c/n_1$ and $v_2 = c/n_2$, show that

$$E_{0,R} = \left|\frac{n_1-n_2}{n_1+n_2}\right| E_{0,I} \qquad E_{0,T} = \left(\frac{2n_1}{n_1+n_2}\right) E_{0,I}$$

Starting from

$$\tilde{E}_{0,R} = \frac{v_2-v_1}{v_2+v_1}\tilde{E}_{0,I} \Rightarrow E_{0,R} = \left|\frac{v_2-v_1}{v_2+v_1}\right| E_{0,I}$$

sign of $v_2 - v_1$ only affects the phase

$$\frac{v_2-v_1}{v_2+v_1} = \frac{c/n_2 - c/n_1}{c/n_2 + c/n_1}\left(\frac{n_1 n_2}{n_1 n_2}\right) = \frac{n_1-n_2}{n_1+n_2} \checkmark$$

$$\tilde{E}_{0,T} = \frac{2v_2}{v_2+v_1}\tilde{E}_{0,I} \Rightarrow E_{0,T} = \frac{2v_2}{v_2+v_1} E_{0,I}$$

$$\frac{2v_2}{v_2+v_1} = \frac{2c/n_2}{c/n_2 + c/n_1}\,\frac{n_1 n_2}{n_1 n_2} = \frac{2n_1}{n_1+n_2} \checkmark$$