PHYS 334 Electromagnetic Theory II

In Class Exercise 14 — February 21, 2024 **Name:** Solutions

1. (a) Given $\mathbf{k}_I \cdot \mathbf{r} = \mathbf{k}_R \cdot \mathbf{r} = \mathbf{k}_T \cdot \mathbf{r}$, consider a point on the boundary $(x, y, z) = (0, y, 0)$. What does this equation become? If this has to hold for all values of y, what does this imply?

$$k_{I,y}\, y = k_{R,y}\, y = k_{T,y}\, y \qquad \text{valid for all } y$$

$$\Rightarrow \boxed{k_{I,y} = k_{R,y} = k_{T,y}}$$

(b) If you were to consider a point on the boundary $(x, 0, 0)$ and require it to hold for all x, what would this imply?

$$\boxed{k_{I,x} = k_{R,x} = k_{T,x}}$$

2. Write $k_{I,x} = k_{R,x} = k_{T,x}$ in terms of the magnitudes k_I, k_R, and k_T and the angles θ_I, θ_R, and θ_T. Given that $k_R = k_I$ and $k_T = \frac{n_2}{n_1} k_I$, what can you conclude?

$k_{I,x} = k_I \sin\theta_I$ $\quad$ $k_{R,x} = k_R \sin\theta_R$ $\quad$ $k_{T,x} = k_T \sin\theta_T$

Since $k_I = k_R \Rightarrow$ $\cos\theta_I = \cos\theta_R \Rightarrow$ $\boxed{\theta_R = \theta_I}$

$k_T = \frac{n_2}{n_1} k_I \Rightarrow$

$k_{T,x} = k_T \sin\theta_T = \frac{n_2}{n_1} k_I \sin\theta_T$

$k_{I,x} = k_I \sin\theta_I$

$\frac{n_2}{n_1} \sin\theta_T = \sin\theta_I$

or $\boxed{n_1 \sin\theta_I = n_2 \sin\theta_T}$