PHYS 334 Electromagnetic Theory II

In Class Exercise 17 — February 28, 2024 Name: Solutions

1. Starting from

$$\tilde{k} = \frac{\omega}{c}\sqrt{\epsilon_r} = \frac{\omega}{c}\left[1 + \frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}\right] = k + i\kappa$$

show that the real part gives

$$n = \frac{ck}{\omega} = 1 + \frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j(\omega_j^2 - \omega^2)}{(\omega_j^2 - \omega^2)^2 + \gamma_j^2\omega^2}$$

and the imaginary part gives

$$\alpha = 2\kappa = \frac{Nq^2\omega^2}{m\epsilon_0 c}\sum_j \frac{f_j\gamma_j}{(\omega_j^2 - \omega^2) + \gamma_j^2\omega^2}$$

~~Re~~ $\frac{1}{a-ib} = \frac{a+ib}{a^2+b^2} \Rightarrow Re\left(\frac{1}{a-ib}\right) = \frac{a}{a^2+b^2}$ $\quad Im\left(\frac{1}{a-ib}\right) = \frac{b}{a^2+b^2}$

$$\Rightarrow k = \frac{\omega}{c}\left[1 + \frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j \overbrace{(\omega_j^2-\omega^2)}^{a}}{(\omega_j^2-\omega^2)^2 + (\gamma_j\omega)^2}\right]$$

and $n = \frac{ck}{\omega} = [$ same $]$ ✓

$$\alpha = 2\kappa = 2\frac{\omega}{c}\left[\frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j\gamma_j\omega}{(\omega_j^2-\omega^2)^2 + (\gamma_j\omega)^2}\right] = \frac{Nq^2\omega^2}{m\epsilon_0 c}\sum_j \frac{f_j\gamma_j}{(\omega_j^2-\omega^2)^2 + (\gamma_j\omega)^2}$$

✓

2. Given $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$, express Gauss's law in terms of V and $\mathbf{A}$.

$$\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \left(-\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}\right)$$

$$= \boxed{-\nabla^2 V - \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = \rho/\varepsilon_0}$$

3. Show that the gauge transformation $\mathbf{A}' = \mathbf{A} + \nabla\lambda$ and $V' = V - \frac{\partial \lambda}{\partial t}$ leaves $\mathbf{E}$ unchanged for any choice of λ.

$$\vec{E} = -\vec{\nabla} V' - \frac{\partial \vec{A}'}{\partial t}$$

$$= -\vec{\nabla}\left(V - \frac{\partial \lambda}{\partial t}\right) - \frac{\partial}{\partial t}\left(\vec{A} + \vec{\nabla}\lambda\right)$$

$$= -\vec{\nabla} V + \vec{\nabla} \cancel{\frac{\partial \lambda}{\partial t}} - \frac{\partial \vec{A}}{\partial t} - \cancel{\frac{\partial}{\partial t} \vec{\nabla}\lambda}$$

$$= -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t} \quad \checkmark$$