PHYS 334 Electromagnetic Theory II

In Class Exercise 18 — March 8, 2024 Name: Solutions

1. Apply the Lorentz gauge condition to Gauss's Law to eliminate **A** and derive a differential equation for V only.

Gauss's Law $\nabla^2 V + \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{A}) = -\rho/\varepsilon_0$

Lorentz gauge $\vec{\nabla}\cdot\vec{A} = -\mu_0\varepsilon_0 \frac{\partial V}{\partial t}$

$$\Rightarrow \boxed{\nabla^2 V - \mu_0\varepsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon_0}}$$

2. Given a Green's function $f(\mathbf{r},\mathbf{r}')$ that satisfies $\nabla^2 f(\mathbf{r},\mathbf{r}') = -\delta^{(3)}(\mathbf{r}-\mathbf{r}')$, show that

$$V(\mathbf{r}) = \frac{1}{\epsilon_0}\int \rho(\mathbf{r}')f(\mathbf{r},\mathbf{r}')\,d\tau'$$

is the solution to $\nabla^2 V(\mathbf{r}) = -\rho(\mathbf{r})/\epsilon_0$.

$$\nabla^2 V(\vec{r}) = \frac{1}{\varepsilon_0}\int \rho(\vec{r}')\,\nabla^2 f(\vec{r},\vec{r}')\,d\tau'$$ because ∇^2 only acts on $\vec{r}$, not $\vec{r}'$

$$= \frac{1}{\varepsilon_0}\int \rho(\vec{r}')\left[-\delta^{(3)}(\vec{r}-\vec{r}')\right]d\tau'$$

$$= -\frac{\rho(\vec{r})}{\varepsilon_0}$$ δ-fns kill integrals