PHYS 334 Electromagnetic Theory II

In Class Exercise 19 — March 18, 2024 **Name:** Key

1. Given the Green's function for the d'Alembertian,

$$\Box^2 f = -\delta^{(3)}(\mathbf{r}-\mathbf{r}')\delta(t-t') \quad \Rightarrow \quad f(\mathbf{r},\mathbf{r}',t,t') = \frac{1}{4\pi \scriptr}\delta(t-t'-\scriptr/c)$$

show that the solution of $\Box^2 V(\mathbf{r},t) = -\rho(\mathbf{r},t)/\epsilon_0$ is

using given f

$$V(\mathbf{r},t) = \int \frac{\rho(\mathbf{r}', t-\scriptr/c)}{4\pi\epsilon_0 \scriptr} d\tau'$$

$$V(\vec{r},t) = \frac{1}{\varepsilon_0}\int \rho(\vec{r}',t') f(\vec{r},\vec{r}',t,t')\, d\tau'\, dt'$$

$$\Rightarrow \Box^2 V(\vec{r},t) = \frac{1}{\varepsilon_0}\int \rho(\vec{r}',t') \underbrace{\Box^2 f(\vec{r},\vec{r}',t,t')}_{= -\delta^{(3)}(\vec{r}-\vec{r}')\delta(t-t')} d\tau'\, dt'$$

$$= -\frac{\rho(\vec{r},t)}{\varepsilon_0} \checkmark$$

2. Show that $\nabla\rho(\mathbf{r}', t-\scriptr/c) = -\frac{1}{c}\dot{\rho}\hat{\scriptr}$

$$\vec{\nabla}\rho(\vec{r}', t-\scriptr/c) = \dot{\rho}\,\vec{\nabla}(t-\scriptr/c) = -\frac{1}{c}\dot{\rho}\,\vec{\nabla}\scriptr$$

$$= -\frac{1}{c}\dot{\rho}\,\hat{\scriptr} \checkmark$$

3. Show that $[\nabla \times \mathbf{J}(\mathbf{r}', t - \mathscr{r}/c)]_x = \frac{1}{c}[\dot{\mathbf{J}} \times \nabla \mathscr{r}]_x$.

$$[\vec{\nabla} \times \vec{J}]_x = \frac{\partial J_z}{\partial y} - \frac{\partial J_y}{\partial z}$$

$$\frac{\partial J_z}{\partial y} = \dot{J}_z \frac{\partial}{\partial y}\left(t - \frac{\mathscr{r}}{c}\right) = -\frac{1}{c} \dot{J}_z \frac{\partial \mathscr{r}}{\partial y}$$

$$\frac{\partial J_y}{\partial z} = -\frac{1}{c} \dot{J}_y \frac{\partial \mathscr{r}}{\partial z}$$

$$\Rightarrow [\vec{\nabla} \times \vec{J}]_x = -\frac{1}{c}\left[(\vec{\nabla}\mathscr{r}) \times \dot{\vec{J}}\right]_x = +\frac{1}{c}\left[\dot{\vec{J}} \times \vec{\nabla}\mathscr{r}\right]_x$$