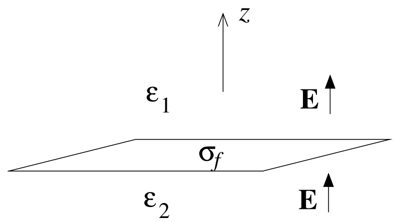
Physics 334 **Electromagnetic Theory II** **Spring 2024**

Review Worksheet for Exam 1

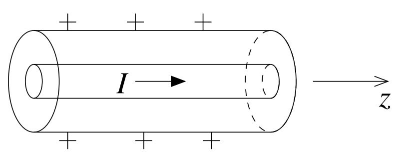
1. Starting from Maxwell's equations, derive the continuity equation for charge conservation: $\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot \mathbf{J}$.

2. A free surface charge density σ_f is distributed along the x-y plane. Above the plane is a linear medium with dielectric constant ϵ_1 and below the plane is a linear medium with dielectric constant ϵ_2.



The free charge density is exactly the right amount such that there is no discontinuity in the electric field, that is, $\mathbf{E} = E_0\,\hat{\mathbf{z}}$ everywhere in space. Find σ_f.

3. Two concentric cylinders have radii a and b, with $a < b$. The inner cylinder carries a current I in the $\hat{\mathbf{z}}$ direction along the surface of the cylinder. The outer cylinder contains along its surface a charge λ per unit length.



   (a) Find the $\mathbf{E}$ and $\mathbf{B}$ fields in each of three regions (i) $s < a$, (ii) $a < s < b$, and (iii) $s > b$.

   (b) Find the Poynting vector in each of the three regions.

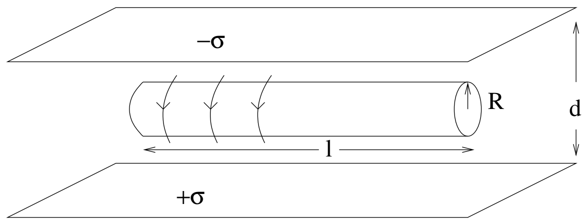
   (c) Find the electromagnetic energy stored in the region $a < s < b$.

4. The Maxwell stress tensor is

$$T_{ij} = \frac{1}{2}\epsilon 0\left(E_iE_j - \frac{1}{2}\delta_{ij}E^2\right) + \frac{1}{2\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$$

Consider a solenoid with magnetic field $\mathbf{B} = \mu_0 nI\,\hat{\mathbf{z}}$ inside. Find all nine elements of the stress tensor for the region inside the solenoid, and express your answer in matrix form.

5. A parallel plate capacitor with separation d has charge densities σ and $-\sigma$ as shown. Inside the plates is a solenoid with radius R, carrying current I through N turns per unit length. The solenoid has length $\ell \gg R$.



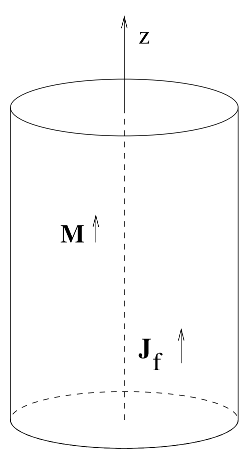
(a) Find $\mathbf{E}$ and $\mathbf{B}$ everywhere (specify each region as necessary). Assume that the parallel plates extend to infinity, and the fringing of the $\mathbf{B}$ field at the ends of the solenoid is negligible.

(b) Find the Poynting vector $\mathbf{S}$.

(c) Calculate the **total** momentum $\mathbf{p}$ for a length L of the solenoid.

(d) If the current is gradually turned off, what force per length (magnitude and direction) is needed to keep the solenoid in place? Use $\mathbf{F} = d\mathbf{p}/dt$. (Any guesses as to where the opposing force comes from?)

6. Consider an infinite cylinder of radius R, centered along the z-axis as shown. The interior of the cylinder is magnetized with field $\mathbf{M} = M\,\hat{\mathbf{z}}$, and a uniform free current density $\mathbf{J}_f = J_f\,\hat{\mathbf{z}}$ flows.



(a) Use Ampere's equation in matter to find $\mathbf{H}$ in the cylinder.

(b) Find $\mathbf{B}$ in the cylinder.