Physics 334 **Electromagnetic Theory II** **Spring 2024**

Review Worksheet for Exam 2

1. (a) Starting from Maxwell's Equations in a vacuum, with $\rho = 0$ and $\mathbf{J} = 0$, derive the wave equation for $\mathbf{E}$.

   (b) For the solution $\tilde{\mathbf{E}} = \tilde{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\,\hat{\mathbf{n}}$ use Maxwell's equations to show that the wave is transverse, that is, $\hat{\mathbf{n}} \cdot \hat{\mathbf{k}} = 0$.

2. For the wave

$$\tilde{\mathbf{f}} = \tilde{A}e^{i(kz-\omega t)}\,\hat{\mathbf{x}} + \tilde{A}e^{i(kz-\omega t+\delta_y)}\,\hat{\mathbf{y}}$$

   with a general complex amplitude $\tilde{A}$ and the following values of δ_y, find $\mathbf{f} = \mathrm{Re}\,\tilde{\mathbf{f}}$ and identify whether the wave is linearly polarized, circularly polarized, or neither.

   The following trig identities might be helpful: $\cos(\theta + 90°) = -\sin(\theta)$ and $\cos(\theta + 180°) = -\cos(\theta)$.

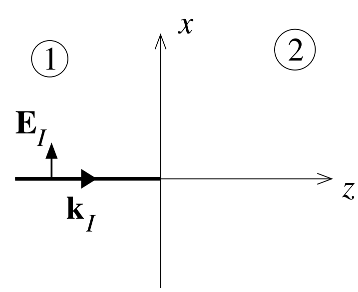
   (a) $\delta_y = 90°$

   (b) $\delta_y = 180°$

3. For a boundary between linear media in the absence of free charge or current, the boundary conditions are

$$\text{(i) } \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}, \qquad \text{(ii) } B_1^{\perp} = B_2^{\perp}, \qquad \text{(iii) } \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel}, \qquad \text{(iv) } \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel}$$

   Consider an EM wave $\tilde{\mathbf{E}}_I = \tilde{E}_{0_I} e^{i(\mathbf{k}_I\cdot\mathbf{r}-\omega t)}\,\hat{\mathbf{x}}$ approaching an interface at normal incidence, as shown below



   (a) Apply each of the boundary conditions (i)–(iv) and express the resulting equation in terms of $\tilde{E}_{0_I}$, $\tilde{E}_{0_R}$, and $\tilde{E}_{0_T}$.

   (b) Solve the resulting equations for $\tilde{E}_{0_R}$ and $\tilde{E}_{0_T}$.

4. Given the potentials

$$V(\mathbf{r},t) = 0 \qquad \mathbf{A}(\mathbf{r},t) = \begin{cases} kts\ \hat{\boldsymbol{\phi}} & s < R \\ kt\dfrac{R^2}{s}\ \hat{\boldsymbol{\phi}} & s \geq R \end{cases}$$

(a) Find the $\mathbf{E}$ and $\mathbf{B}$ fields everywhere.

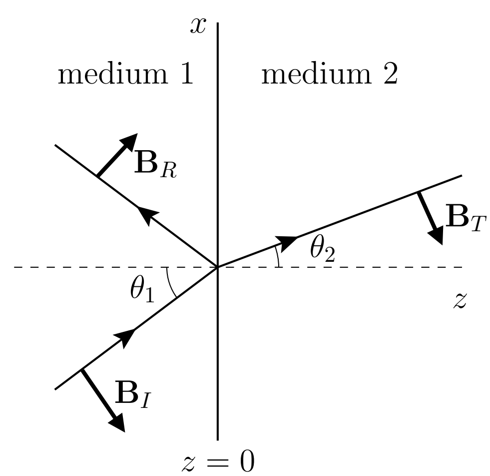
(b) Based on your answer to (b), what physical situation does this describe?

5. For a hydrogen gas the complex wavevector is given by

$$\tilde{k} \simeq \frac{\omega}{c}\left[1 + \frac{Nq^2}{2m\epsilon_0}\left(\frac{1}{\omega_0^2 - \omega^2 - i\gamma\omega}\right)\right]$$

with $\gamma \ll \omega$ and $\gamma \ll \omega_0$. Argue why this gives an absorption coefficient $\alpha = 2\kappa$ that as a function of ω is sharply peaked around ω_0.

6. Consider a plane wave at oblique incidence to an interface located at $z = 0$. The wave has plane polarization perpendicular to the plane of incidence, with the electric field vectors $\mathbf{E}_I$, $\mathbf{E}_R$, and $\mathbf{E}_T$ all pointing out of the page. (Medium 1 has ϵ_1 and μ_1, and medium 2 has ϵ_2 and μ_2.)



(a) Write down the equation for $\mathbf{E}_\parallel$ at the boundary. Derive *two* equations by letting x and y vary independently along the interface.

(b) From these two equations, derive Snell's Law for the angle of refraction.