Physics 334 **Electromagnetic Theory II** **Spring 2024**

Review Worksheet for Exam 3

1. An electric dipole oscillating in the $\hat{\mathbf{z}}$ direction creates the potentials

$$V(r,\theta,t) = -\frac{p_0\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin[\omega(t-r/c)]$$
$$\mathbf{A}(r,\theta,t) = -\frac{\mu_0 p_0\omega}{4\pi r}\sin[\omega(t-r/c)]\,\hat{\mathbf{z}}$$

   Calculate the radiation parts of $\mathbf{E}$ and $\mathbf{B}$.

2. Alfred is travelling in a train from Lewisburg to Harrisburg at speed $v = 0.6c$. Beatrice is standing on the platform watching Alfred go by, and she notices that his watch is running slow. Alfred also notices Beatrice's station clock is running slow. Their disagreement is threatening to ruin their marriage, so they decide to set up a test to see who is right.

   Beatrice synchronizes a set of clocks along the tracks between Lewisburg and Harrisburg, and Alfred sets his watch exactly as he passes Lewisburg to match Beatrice's clock. When Alfred arrives at Harrisburg, Beatrice points out that his watch is reading an earlier time than her Harrisburg clock, and she says "Aha, I was right! Your watch is running slow." Alfred agrees that his watch reads an earlier time, but disagrees that it is running slow.

   In order to salvage their marriage, explain what is going on and why both are right.

3. For a 4-vector a^μ transformed from frame S to frame $\bar{S}$, show $\bar{a}_\mu \bar{a}^\mu = a_\mu a^\mu$.

4. The potentials for an arbitary moving charge in the Lorentz gauge are

$$V(\mathbf{r},t) = \frac{q}{4\pi\epsilon_0 \mathscr{r}}\frac{1}{1-\hat{\mathscr{r}}\cdot\mathbf{v}} \qquad \mathbf{A}(\mathbf{r},t) = \frac{\mathbf{v}}{c^2}V(\mathbf{r},t)$$

   where $\mathscr{r} = \mathbf{r} - \mathbf{w}(t_r)$ is the difference between the observation point and the retarded position of the particle, and $\mathbf{v}$ is evaluated at retarded time.

   (a) In *words* (no equations allowed!), explain why $\mathscr{r}$ and $\mathbf{v}$ are evaluated at retarded time, rather than time t.

   (b) For constant velocity $\mathbf{v} = v\,\hat{\mathbf{z}}$, find t_r at a point $(\mathbf{r},t)$.

5. The electric field of a point charge in motion is given by

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \frac{\mathcal{r}}{(\boldsymbol{\mathcal{r}} \cdot \mathbf{u})^3} [\mathbf{u}(c^2 - v^2) + \boldsymbol{\mathcal{r}} \times (\mathbf{u} \times \mathbf{a})]$$

where $\mathbf{u} = c\hat{\boldsymbol{\mathcal{r}}} - \mathbf{v}$, and the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ are evaluated at the retarded time. A charge q undergoes uniform circular motion in the x-y plane:

$$\mathbf{w}(t) = R \cos \omega t\, \hat{\mathbf{x}} + R \sin \omega t\, \hat{\mathbf{y}}$$

(a) For a point along the z-axis a distance h from the origin, find the retarded time t_r.

(b) Find $\boldsymbol{\mathcal{r}} = \mathbf{r} - \mathbf{w}(t_r)$, $\mathbf{v}(t_r)$ and $\mathbf{a}(t_r)$ (express your answers in terms of $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$, and t, NOT t_r).

(c) Use these to obtain the *radiation* part of the $\mathbf{E}$ field. Simplify your answer as much as possible.

6. Given the Poynting vector for radiation from a point charge,

$$\mathbf{S}_{rad} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \left(\frac{\sin^2 \theta}{\mathcal{r}^2} \right) \hat{\boldsymbol{\mathcal{r}}}$$

derive the Larmor formula $P = \dfrac{\mu_0 q^2 a^2}{6\pi c}$.