Reading Assignments for Weeks 5 & 6

- Friday, February 14: Eq. (5.35) through the Math Behind the Models: Stirling Approximation (pp. 221–222), Section 6.1 up to the beginning of 6.1.1 (pp. 237–241), Section 6.1.3 (pp. 248–250), Math Behind the Models: the Gaussian Integral (p. 261).
- Monday, February 17: Sections 6.1.1 and 6.1.2 (pp. 241–248).
- Wednesday, February 19: Sections 6.2.2 and 6.2.3 (pp. 262–267).
- Friday, February 21: Sections 6.3, 6.4, and 6.5 (pp. 267–276).

Homework #4 — due Friday, February 21

1. Problem 5.6
2. **Problem G. The Boltzmann Factor from the Einstein Solid.** Recall from PHYS 211 the Einstein solid model, which had N oscillators and q energy units, and the multiplicity given by

$$W(N,q) = \frac{(q+N-1)!}{q!(N-1)!} \approx \frac{(q+N)!}{q!N!}$$

The simpler, approximate form is valid whe $N \gg 1$. The energy units are of size ϵ, so the energy E is related to the number of energy units q by the relation $E = q\epsilon$.

(a) For an Einstein solid reservoir (large N), show that the temperature definition

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_N$$

leads to the result

$$\frac{q}{q+N} = e^{-\epsilon/k_B T}$$

(b) Now consider a single oscillator with q_1 energy units. Show that the multiplicity $W = 1$ regardless of q_1.

(c) Now consider the single oscillator and the N oscillator reservoir as two coupled systems that can exchange energy. The single oscillator has q_1 energy units and the reservoir has $q - q_1$ units. Assume $q_1 \ll q$. Show that

$$\frac{\text{Prob(single oscillator has } q_1 \text{ units)}}{\text{Prob(single oscillator has 0 units)}} = \left(\frac{q}{q+N}\right)^{q_1}$$

(d) Combine your results from parts (a) and (c) to argue that the probability that the single oscillator has energy E_1 is given by

$$P(E_1) \propto e^{-E_1/k_B T}$$

and now you've derived the Boltzmann factor for the Einstein solid!

3. Problem 6.1
4. Problem 6.2
5. **Problem H. Plotting Two-State Systems**
   (a) Reproduce the plots of the Hill function shown in Fig. 6.6 (p. 244).
   (b) Reproduce the plot of the probability of promoter occupancy shown in Fig. 6.13 (p. 248).
6. Problem 6.8
7. Problem 6.9