Equations for Final Exam

Unit 1

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \qquad -\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \left[V + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u = Eu \quad \text{[for } u(r) = rR(r)\text{]}$$

$$\int_0^{2\pi}\int_0^{\pi} Y^*_{lm}(\theta,\phi)\, Y_{l'm'}(\theta,\phi)\sin\theta\, d\theta\, d\phi = \delta_{ll'}\delta mm'$$

$$V_H(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r} \qquad E_n^H = -\frac{\hbar^2}{2ma^2}\frac{1}{n^2} \qquad a = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.529\times 10^{-10}\ \text{m}$$

$$[L_x, L_y] = i\hbar L_z \qquad [L_y, L_z] = i\hbar L_x \qquad [L_z, L_x] = i\hbar L_y \qquad L_\pm = L_x \pm iL_y \quad \text{(same for } \mathbf{J}, \mathbf{S}, \text{etc)}$$

$$L_\pm|l\,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm 1)}\,|l\,(m\pm 1)\rangle$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$|\psi_n^{(1)}\rangle = \sum_{m\neq n}\frac{\langle\psi_m^{(0)}|H'|\psi_n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}\,|\psi_m^{(0)}\rangle \qquad E_n^{(2)} = \sum_{m\neq n}\frac{|\langle\psi_m^{(0)}|H'|\psi_n^{(0)}\rangle|^2}{E_n^{(0)} - E_m^{(0)}}$$

$$\langle\mathbf{L} + 2\mathbf{S}\rangle = g_J\langle\mathbf{J}\rangle \qquad g_J = 1 + \frac{j(j+1) - l(l+1) + 3/4}{2j(j+1)}$$

Unit 2

$$\Psi(t) = c_a(t)\psi_a e^{-iE_at/\hbar} + c_b(t)\psi_b e^{-iE_bt/\hbar} \qquad \dot{c}_a = -\frac{i}{\hbar}H'_{ab}e^{-i\omega_0 t}c_b \qquad \dot{c}_b = -\frac{i}{\hbar}H'_{ba}e^{i\omega_0 t}c_a$$

$$\omega_0 = \frac{E_b - E_a}{\hbar} \qquad P_{a\to b}(t) \simeq \frac{|V_{ab}|^2}{\hbar^2}\frac{\sin^2[(\omega_0 - \omega)t/2]}{(\omega - \omega_0)^2} \qquad R_{b\to a} = \frac{\pi}{3\epsilon_0\hbar^2}|p|^2\rho(\omega_0)$$

$$\psi = A\left\{e^{ikz} + f(\theta)\frac{e^{ikr}}{r}\right\} \qquad D(\theta) = \frac{d\sigma}{d\Omega} = |f(\theta)|^2 \qquad \sigma = \int D(\theta)d\Omega \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

$$\psi = A\left\{e^{ikz} + k\sum_{l=0}^{\infty} i^{l+1}(2l+1)a_l h_l^{(1)}(kr)P_l(\cos\theta)\right\} \qquad f(\theta) = \sum_{l=0}^{\infty}(2l+1)a_l P_l(\cos\theta)$$

$$e^{ikz} = \sum_{l=0}^{\infty} i^l(2l+1)j_l(kr)P_l(\cos\theta) \qquad P_0(\cos\theta) = 1 \qquad P_1(\cos\theta) = \cos\theta \qquad P_2(\cos\theta) = \tfrac{3}{2}\cos^2\theta - \tfrac{1}{2}$$

$$f(\theta,\phi) \simeq -\frac{m}{2\pi\hbar^2}\int e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}_0}V(\mathbf{r}_0)d^3\mathbf{r}_0 \qquad f(\theta) \simeq -\frac{2m}{\hbar^2\kappa}\int_0^\infty rV(r)\sin(\kappa r)dr \quad \text{with } \kappa = 2k\sin(\theta/2)$$

Unit 3

$$x^{0\prime} = \gamma(x^0 - \beta x^1) \qquad x^{1\prime} = \gamma(x^1 - \beta x^0) \qquad x^{2\prime} = x^2 \qquad x^{3\prime} = x^3 \qquad \beta = v/c \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\eta^\mu = \frac{dx^\mu}{d\tau} = \gamma(c, v_x, v_y, v_z) \qquad \mathbf{p} = \gamma m \mathbf{v} = \frac{m\mathbf{v}}{\sqrt{1 - v^2/c^2}} \qquad E = \gamma mc^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

$$\text{CM 2-body decay: } \Gamma = \frac{S|\mathbf{p}|}{8\pi\hbar m_1^2 c}|\mathcal{M}|^2 \qquad \text{CM 2-body scattering: } \frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1 + E_2)^2}\frac{|\mathbf{p}_f|}{|\mathbf{p}_i|}$$

$$\text{KG: } -\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} + \nabla^2\psi = \left(\frac{mc}{\hbar}\right)^2\psi \qquad \text{Dirac: } i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0 \qquad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^x = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Unit 4

$$u^{(1)}(p) = N\begin{pmatrix} 1 \\ 0 \\ \frac{cp_z}{E + mc^2} \\ \frac{c(p_x + ip_y)}{E + mc^2} \end{pmatrix} \qquad u^{(2)}(p) = N\begin{pmatrix} 0 \\ 1 \\ \frac{c(p_x - ip_y)}{E + mc^2} \\ \frac{-cp_z}{E + mc^2} \end{pmatrix} \qquad N = \sqrt{(E + mc^2)/c}$$

$$v^{(1)}(p) = N\begin{pmatrix} \frac{c(p_x - ip_y)}{E + mc^2} \\ \frac{-cp_z}{E + mc^2} \\ 0 \\ 1 \end{pmatrix} \qquad v^{(2)}(p) = -N\begin{pmatrix} \frac{cp_z}{E + mc^2} \\ \frac{c(p_x + ip_y)}{E + mc^2} \\ 1 \\ 0 \end{pmatrix} \qquad \text{where } E > 0$$

$$\text{electrons: } \psi = ae^{-(i/\hbar)p\cdot x}u^{(s)}(p) \qquad \text{positrons: } \psi = ae^{(i/\hbar)p\cdot x}v^{(s)}(p) \qquad \bar{\psi} = \psi^\dagger\gamma^0$$

$$\psi' = S\psi \qquad S = \begin{pmatrix} a_+ & a_-\sigma_1 \\ a_-\sigma_1 & a_+ \end{pmatrix} \qquad a_+ = \sqrt{(\gamma + 1)/2} \qquad a_- = -\sqrt{(\gamma - 1)/2}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \qquad A^\mu := (V, \mathbf{A}) \qquad J^\mu := (c\rho, \mathbf{J}) \qquad \partial_\mu F^{\mu\nu} = \frac{4\pi}{c}J^\nu$$

Maxwell's Eqs. and the relations between fields and potentials given as needed.

$$\partial_\mu A^\mu = 0 \qquad A^0 = 0 \qquad \Box A^\mu = 0 \qquad \text{photons: } A^\mu = ae^{-(i/\hbar)p\cdot x}\epsilon^\mu(p) \qquad \not{b} \equiv \gamma^\mu b_\mu$$

$$\text{ABC vertex: } -ig \qquad \text{ABC propagator: } \frac{i}{q^2 - m^2c^2}$$

$$\text{QED vertex: } ig_e\gamma^\mu \qquad \text{Propagators: } e^\pm\text{: } \frac{i(\gamma^\mu q_\mu + mc)}{q^2 - m^2c^2} \qquad \text{photons: } \frac{-ig_{\mu\nu}}{q^2}$$