Problem A

The notation in time-dependent perturbation theory differs from that of time-independent perturbation theory, but there are nonetheless similarities. To see this, let's add a λ to the perturbing hamiltonian, i.e.

$$H = H^{(0)} + \lambda H'(t)$$

We can get a perturbative solution to [9.13] by expanding c_a and c_b as

$$c_a(t) = f_a^{(0)} + \lambda f_a^{(1)} + \lambda^2 f_a^{(2)} + \ldots \qquad c_b(t) = f_b^{(0)} + \lambda f_b^{(1)} + \lambda^2 f_b^{(2)} + \ldots$$

(a) Derive an equation for $f_a^{(n+1)}$ and $f_b^{(n+1)}$ in terms of $f_a^{(n)}$ and $f_b^{(n)}$. This is the equation you would use to go to one higher order in perturbation theory.

(b) Show that the perturbative expressions $c_a^{(n)}$ and $c_b^{(n)}$ defined in section 9.1.2 are given by

$$c_a^{(n)} = \sum_{i=0}^{n} f_a^{(i)} \qquad c_b^{(n)} = \sum_{i=0}^{n} f_b^{(i)}$$

when $\lambda = 1$.