Problem B

As you showed in problem 7.18, the inhomogeneous Maxwell's Equations — Eqs. 7.70 (i) and (iv) — can be written as $\partial_\mu F^{\mu\nu} = \frac{4\pi}{c} J^\nu$ for the field tensor $F^{\mu\nu}$ given in Eq. 7.71.

(a) An alternate anti-symmetric 2nd rank field tensor can be written as

$$G^{\mu\nu} := \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}$$

This tensor also transforms according to $G^{\mu\nu\prime} = \Lambda^\mu_\lambda \Lambda^\nu_\delta G^{\lambda\delta}$, that is, this tensor is just as valid as $F^{\mu\nu}$ for expressing the transformation of the electric and magnetic fields under a change of frame.
Show that the *homogeneous* Maxwell's Eqs. 7.70 (ii) and (iii) can be written as $\partial_\mu G^{\mu\nu} = 0$.

(b) Show that $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ with $A^\mu := (V, \mathbf{A})$ is equivalent to the field/potential relations

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{E} = -\nabla V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}$$