Problem D

For systems with a spherically symmetric potential, like hydrogen, the states can be represented as wavefunctions of the form

$$|n\,l\,m\rangle \to \psi_{nlm}(\mathbf{r}r) = R_{nl}(r)Y_{lm}(\theta, \phi)$$

where Y_{lm} are the spherical harmonics and $R_{nl}(r)$ are specific to the potential. In this problem we will use properties of the spherical harmonics and integration over the ϕ coordinate to derive the m selection rules.

(a) Using spherical harmonics, show $\langle n'\,l'\,m'|z|n\,l\,m\rangle = 0$ unless $m = m'$.

(b) Using spherical harmonics, show $\langle n'\,l'\,m'|x|n\,l\,m\rangle = 0$ and $\langle n'\,l'\,m'|y|n\,l\,m\rangle = 0$ unless $m = m' \pm 1$.

Problem E

This is basically a modification of Griffiths's problem 9.14.

(a) Same as Griffiths 9.14a.

(b) You should have found three different states that the $|3\,0\,0\rangle$ state decays to, on the way to the ground state. For each of the three, calculate the decay rate. You may find the integral

$$\int_0^\infty R_{30}(r)R_{21}(r)r^3\,dr = \frac{2^7 3^4\sqrt{2}}{5^6}a$$

helpful.

(c) Same as Griffiths 9.14c.