

Priority Queue ADT

Revised based on textbook
author's notes.

Priority Queues

- Some applications require the use of a queue in which items are assigned a priority.
 - higher priority items are dequeued first.
 - items with equal priority still follow FIFO.

Some Applications

- Operating systems such as Linux use priority queues to manage their jobs (try e.g., the `top` command)
- Simulations use priority queues to manage events to be simulated
- All other FIFO queues, e.g., online shopping queues are special cases of priority queue, that is, time is the priority

The Priority Queue ADT

- A *priority queue* is a queue in which each item is assigned a priority and items with a higher priority are removed before those with lower priority.
 - Integer values are used for the priorities.
 - Smaller integers have a higher priority.

The Operations

- `PriorityQueue()`
- `is_empty()`
- `len()`
- `enqueue(item, priority)`
- `dequeue()`
- `peek()`

Priority Queue Example

- Consider the following code segment:

```
Q = PriorityQueue( 6 )
Q.enqueue( "purple", 5 )
Q.enqueue( "black", 1 )
Q.enqueue( "orange", 3 )
Q.enqueue( "white", 0 )
Q.enqueue( "green", 1 )
Q.enqueue( "yellow", 5 )
```



Priority Queue Implementation

- How should the ADT be implemented.

We must consider:

- A priority must be associated with each item in the queue.
- The next item dequeued is the item with the highest priority.
- If multiple items have the same priority, those must be dequeued in a FIFO order.

Priority Queue Implementation

- There can be many different implementations, we'll consider three here
 - Textbook approach
 - Linked list
 - Bounded array with linked lists

1. Textbook approach

- The priority queue is implemented as a Python list
- The enqueue operation puts the item at the end of the queue (as in our FIFO queue)
- The dequeue operation takes the item with the highest priority off the queue (note: the item could be anywhere in the queue!)

Queue operations

```
def enqueue( self, item, priority ) :
    """Adds the given item to the queue."""
    # Create a new instance of the storage class and append it to the list.
    entry = _PriorityQEntry( item, priority )
    self._qlist.append( entry )

def dequeue( self ) :
    """ Removes and returns the first item in the queue."""
    assert not self.is_empty(), "Cannot dequeue from an empty queue."

    # Find the entry with the highest priority
    top = self.find_top_priority()

    # Remove the entry with the highest priority and return the item.
    entry = self._qlist.pop( top )
    return entry

def peek( self ) :
    """Return the value of the top priority without removing it"""
    assert not self.is_empty(), "Cannot peek from an empty queue."

    top = self.find_top_priority()
    return self._qlist[top].item

def find_top_priority( self ) :
    """Find the entry with the highest priority."""
    highest = self._qlist[0].priority
    high_index = 0
    for i in range( len( self ) ) :
        # See if the ith entry contains a higher priority (smaller integer).
        if self._qlist[i].priority < highest :
            highest = self._qlist[i].priority
            high_index = i

    return high_index
```

Details of find_top_priority()

Basically it is the same process of finding a minimum in a list.

```
def find_top_priority(self):
    highest_index = 0
    highest = self._qlist[highest_index].priority
    for i in range(len(self)):
        if highest > self._qlist[i].priority: # smaller value has higher priority
            highest_index = i
            highest = self._qlist[i].priority
    return highest_index
```

Try the program `testpriorityqueue.py`

Complexity of operations

What is the complexity for dequeue?

$O(n)$

What is the complexity for enqueue?

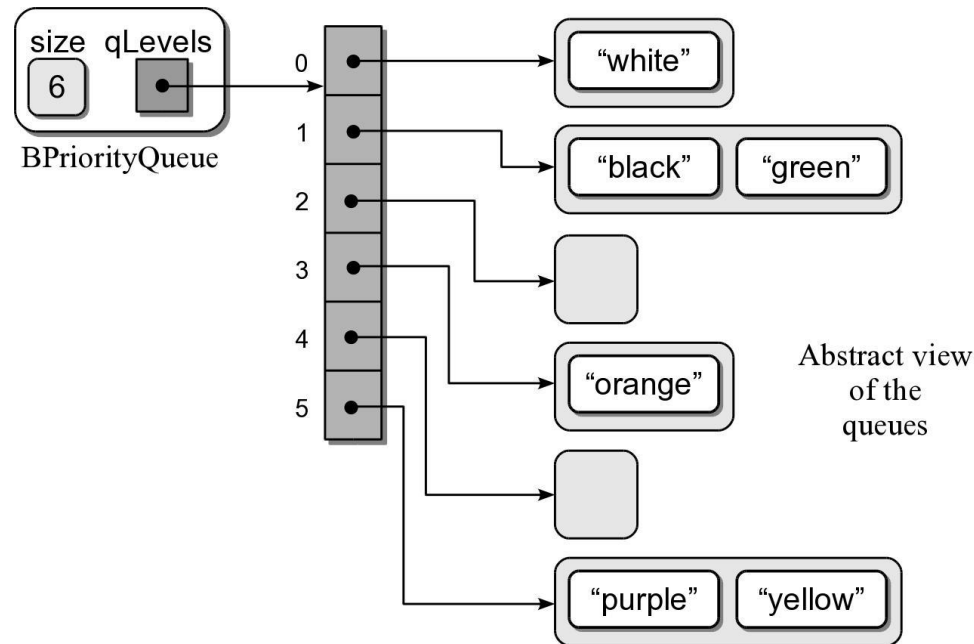
$O(1)$

2. Bounded Priority Queue

- A bounded priority queue has a fixed set of priorities
- We use an array to represent the set of priorities, each array element maintains a queue of the items with the same priority

Bounded Priority Queue

- The following example shows a bounded priority queue with six levels



Bounded Priority Q Implementation

bpriorityq.py

```
from array204 import Array
from llistqueue import Queue

class BPriorityQueue :
    def __init__( self, num_levels = 6 ):
        self._qsize = 0
        self._qlevels = Array( num_levels )
        for i in range( num_levels ) :
            self._qlevels[i] = Queue()

    def is_empty( self ):
        return len( self ) == 0

    def __len__( self ):
        return len( self._qsize )

# ...
```

Bounded Priority Q Implementation

bpriorityq.py

```
class BPriorityQueue :
# ...
    def enqueue( self, item, priority ) :
        assert priority >= 0 and priority < len(self._qlevels), \
            "Invalid priority level."
        self._qlevels[priority].enqueue( item )

    def dequeue( self ) :
        # Make sure the queue is not empty.
        assert not self.is_empty(), "Cannot dequeue from an empty queue."

        # Find the first non-empty queue.
        top_index = self.find_top_priority_queue()

        # We know the queue is not empty, so dequeue from the ith queue.
        return self._qlevels[top_index].dequeue()
```

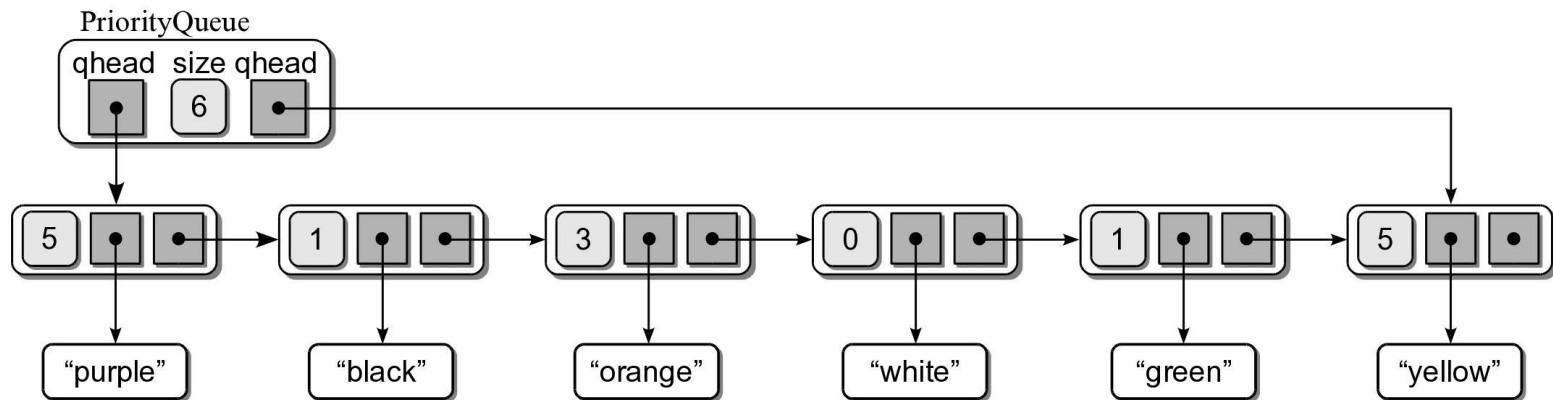

Bounded Priority Q Implementation

bpriorityq.py

```
class BPriorityQueue :  
# ...  
    def find_top_priority_queue(self):  
        # find the first non-empty queue, a.k.a. highest priority  
        i = 0  
        p = len(self._qlevels)  
        while i < p and self._qlevels[i].is_empty() :  
            i += 1  
        return i
```

Unbounded Priority Q: Linked List

- We can use a singly linked list:
 - Head and tail references.
 - Append new entries to the end.



Priority Queue Analysis

- The worst case analysis for the two implementations.

Queue Operation	Python List	Linked List
<code>q = PriorityQueue()</code>	$O(1)$	$O(1)$
<code>len(q)</code>	$O(1)$	$O(1)$
<code>q.is_empty()</code>	$O(1)$	$O(1)$
<code>q.enqueue(x)</code>	$O(n)$	$O(1)$
<code>x = q.dequeue()</code>	$O(n)$	$O(n)$

Implement enqueue()

- Your task is to implement the enqueue() method for a linked list based queue in which other necessary methods have been implemented