Introduction to Binary Trees

Revised based on textbook
author's notes.

Binary Tree ADT

- So far the ADTs we studied are linear
  - Lists
  - Arrays
  - Stacks
  - Queues
- Some applications require non-linear ADTs. Examples may include ADTs
  - To represent an organization
  - To represent class inheritance in OOP
  - To represent a complex algebraic expression
  - Or even to represent a collection of sorted numbers!

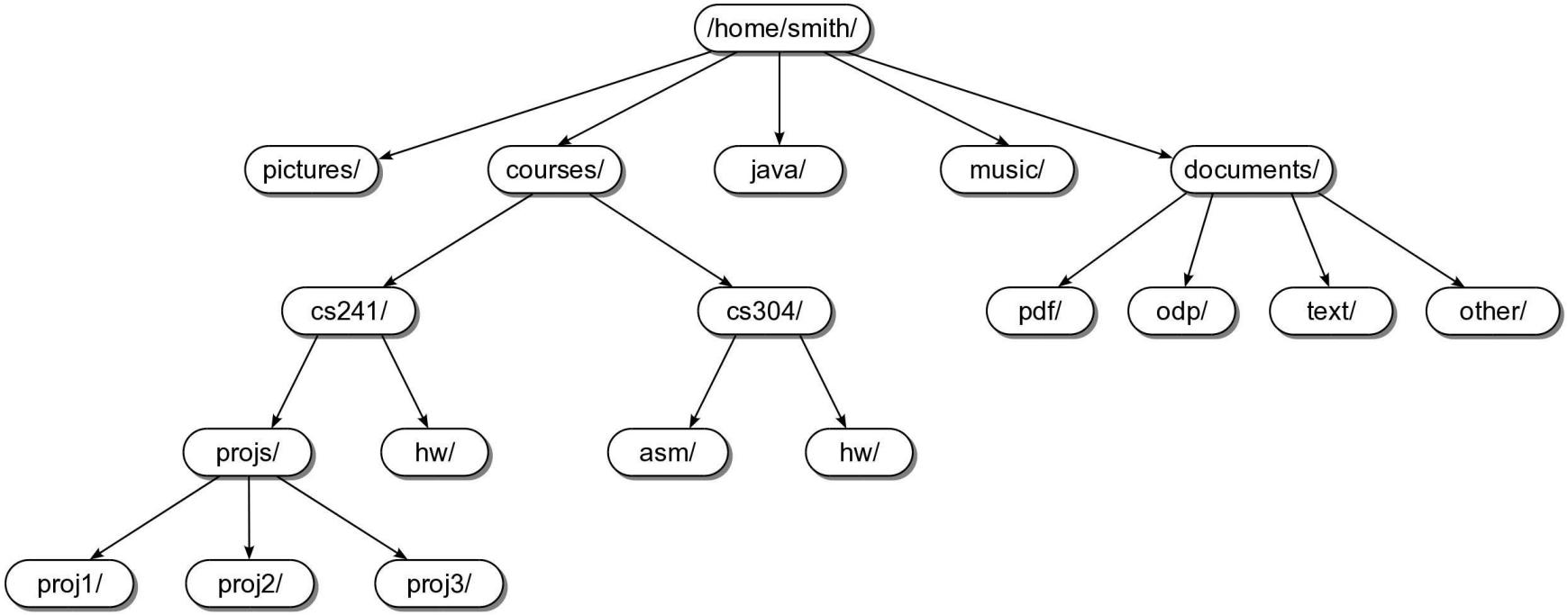
The Tree Structure

- Consists of nodes and edges that organize data in a hierarchical fashion.
  - **nodes** – store the data elements.
  - **edges** – connect the nodes.
- The organization of the nodes form relationships between the data elements.

Definition of a tree

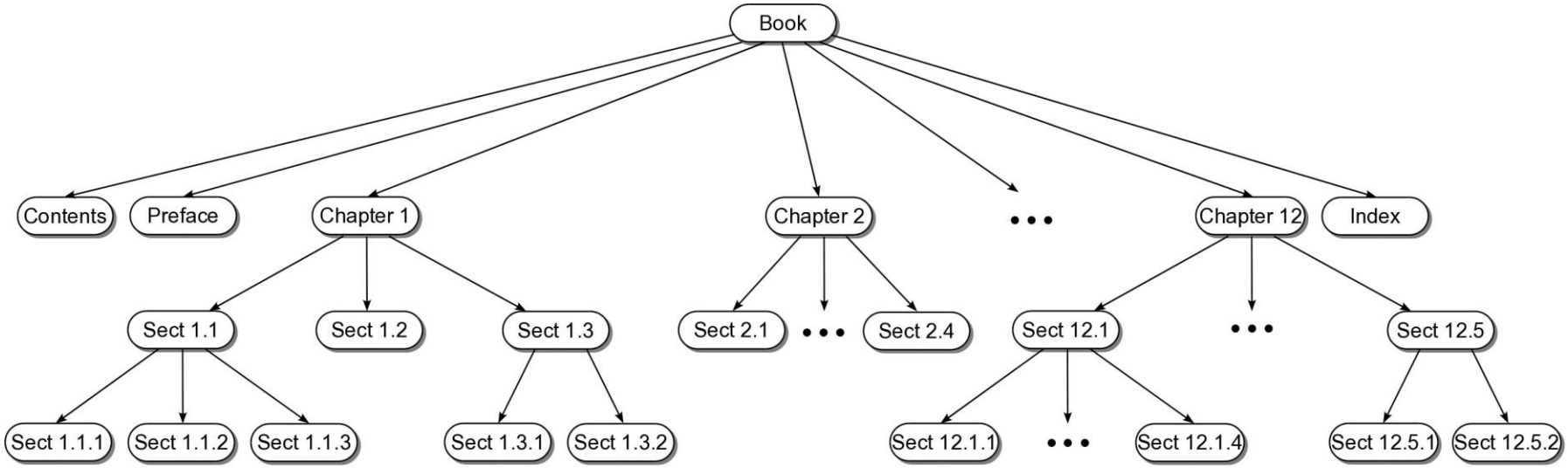
- A tree data structure is defined as follows.
  - A tree consists of a node called root
  - The root may have zero or more children
  - Each children itself is a tree
- A recursive definition!

Tree Example #1



Linux file system is a tree!

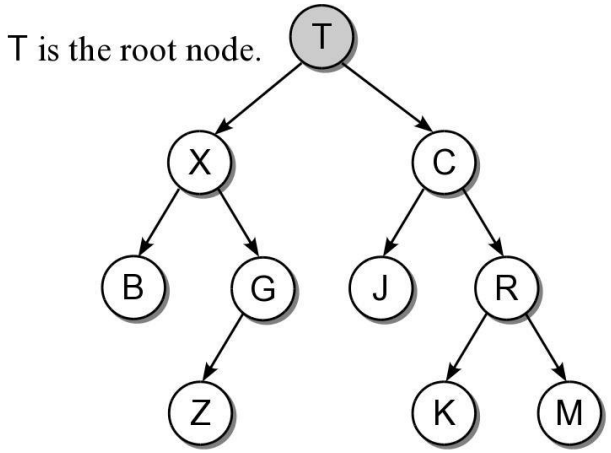
Tree Example #2



Chapters and sections in a book can be organized as a tree!

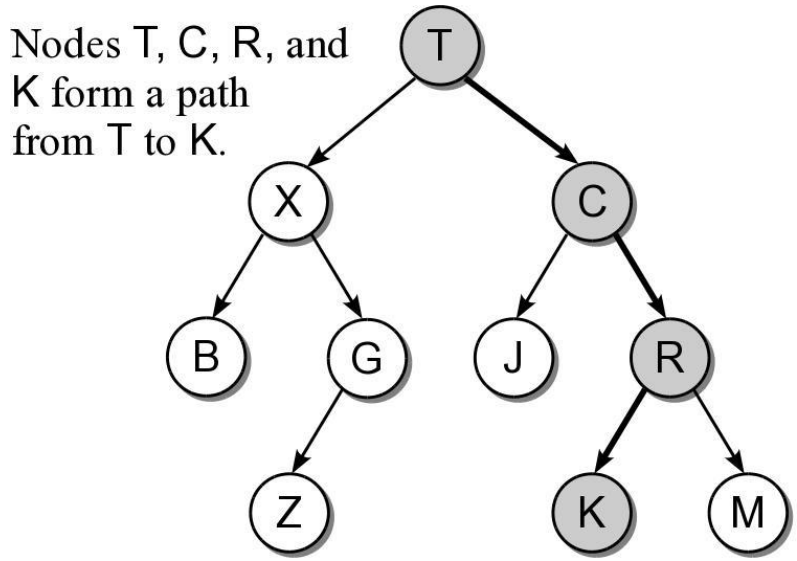
Root Node

- Topmost node of the tree.
  - Provides the single access point into the tree.
  - Has no incoming edges.



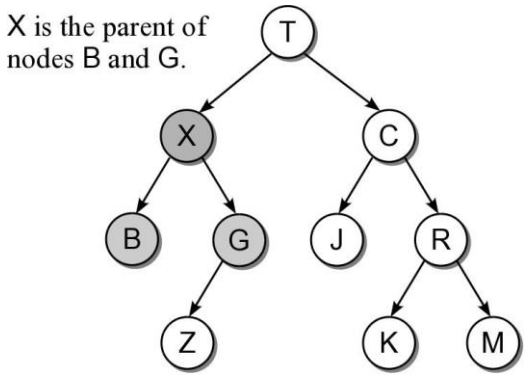
Tree Path

- The nodes encountered when following the edges from the root node to the destination node.
  - Access to all other nodes must start with the root.



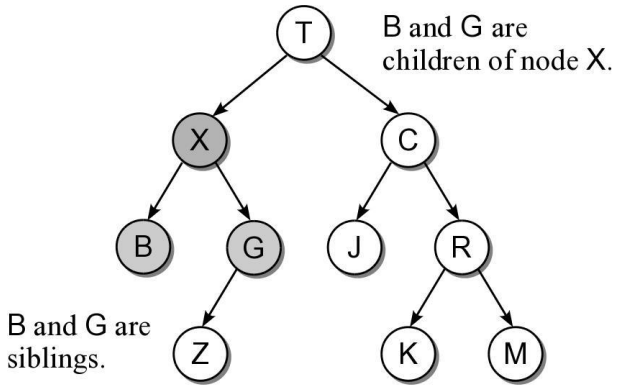
Parent Node

- The node from which an incoming edge originates.
  - Every node, except the root, has a parent node.
  - A node can only have one parent.



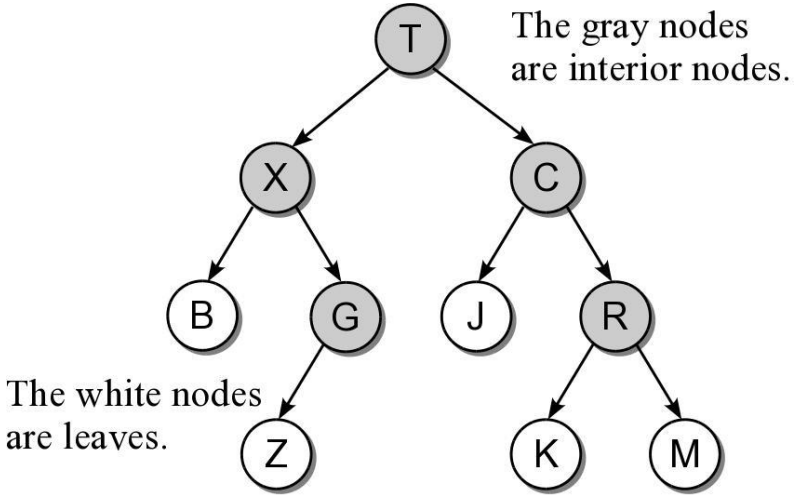
Child Node

- The nodes to which outgoing edges are connected.
  - Each node can have one or more child nodes.
  - Results in a parent-child relationship.
  - **sibling nodes** – all nodes that have the same parent.



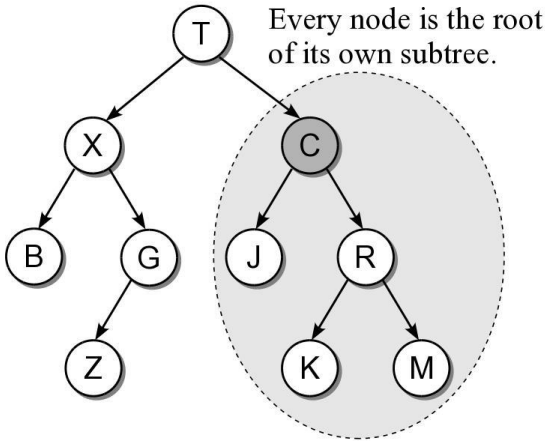
Types of Nodes

- Nodes can be classified as either:
  - **interior node** – a node that has at least one child.
  - **leaf node** – a node that has no children.



Subtree

- A tree is by definition a recursive structure.
  - Every node can be the root of its own subtree.
  - A **subtree** consists of a subset of nodes and edges of the larger tree.

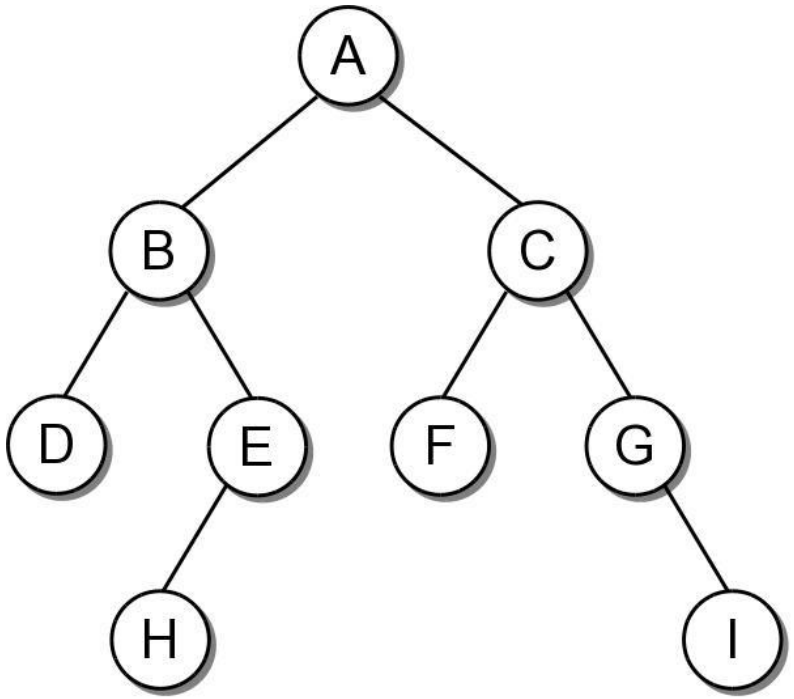


Relatives

- **descendants**
  - All nodes of a subtree are the descendants of the subtree's root.
  - Every node in the tree is a descendant of the root.
- **ancestors**
  - The ancestors of a node include all of the nodes along the node's path, excluding the node itself.
  - The root is the ancestor of all the other nodes.

The Binary Tree

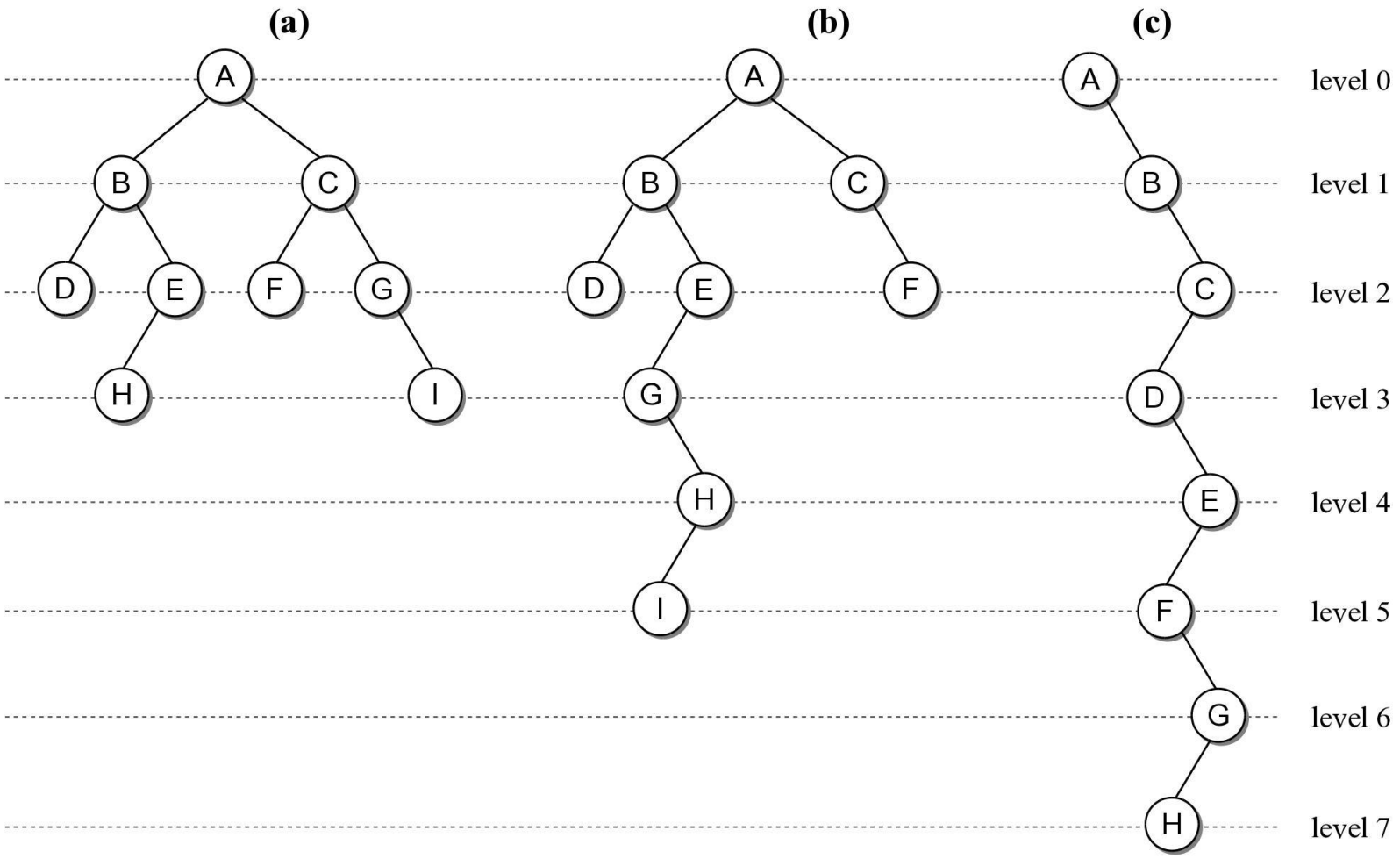
- A tree in which each node can have at most two children. The nodes are commonly labeled:
  - left child
  - right child



Binary Tree Properties

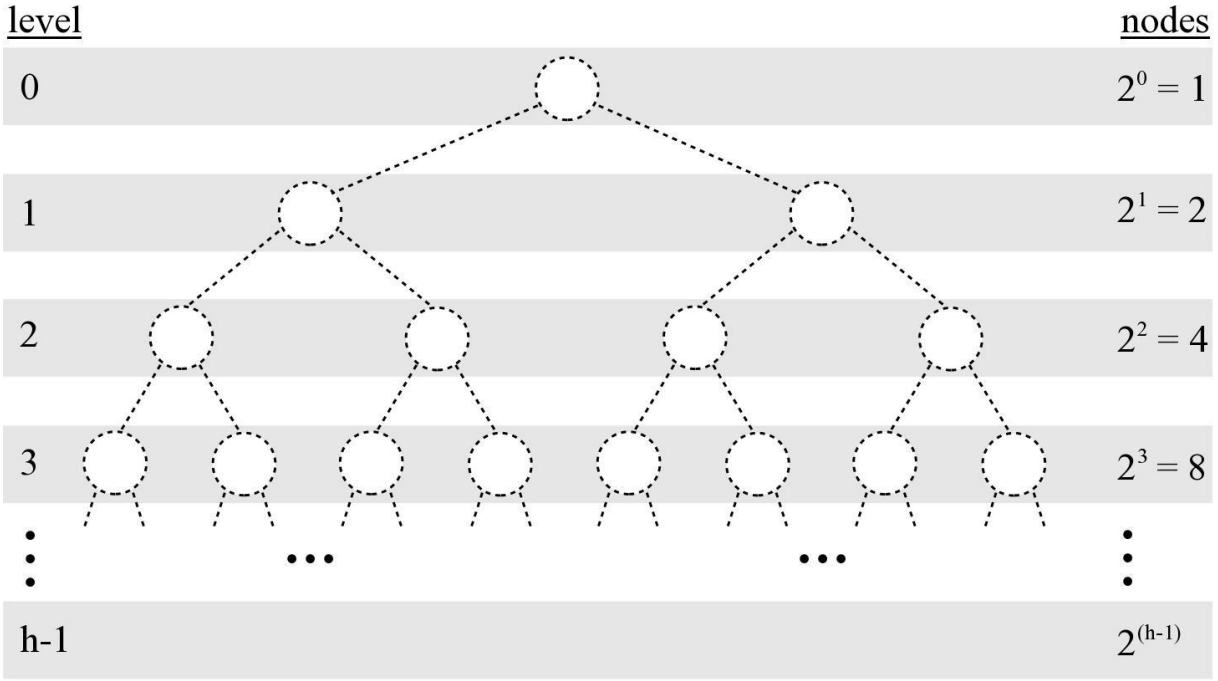
- There are several properties associated with binary trees that depend on the node organization.
  - **depth –** the distance of a node from the root.
  - **level –** all nodes at a given depth share a level.
  - **height –** number of levels in the tree.
  - **width –** number of nodes on the level containing the most nodes.
  - **size –** number of nodes in the tree.

Binary Tree Properties



Binary Tree Properties

- Given a tree of size n:
  - max height = n
  - min heigh $\lfloor \log_2 n \rfloor + 1$

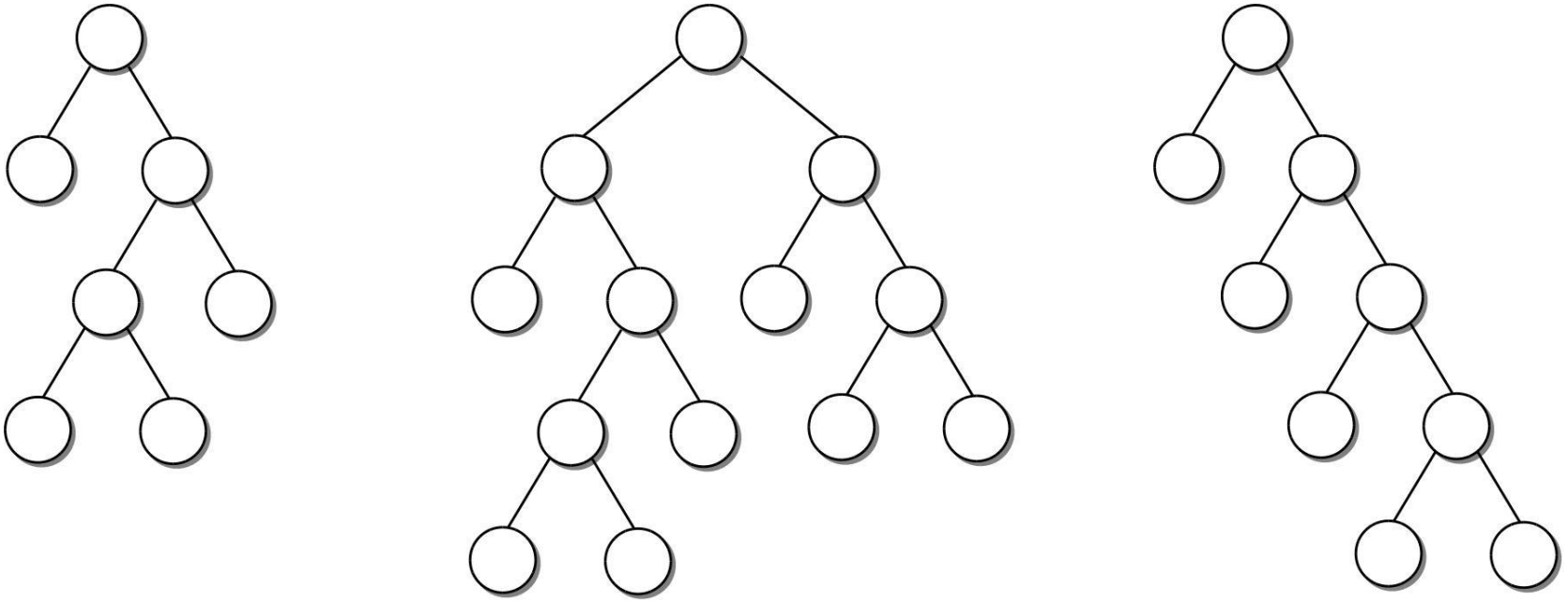


Binary Tree Structure

- Height of a tree will be important in analyzing the efficiency of binary tree algorithms.
- Structural properties can play a role in the efficiency of an algorithm.

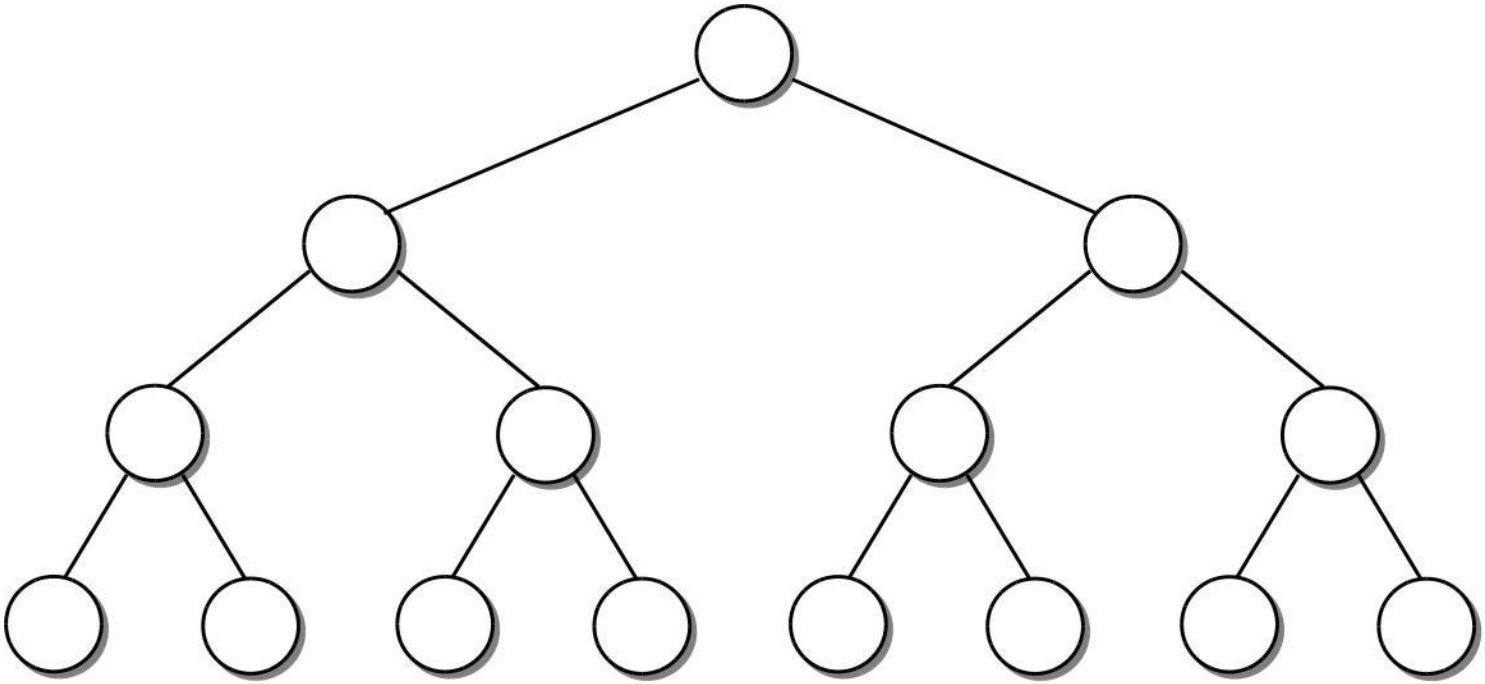
Full Binary Tree

- A binary tree in which each interior node contains two children.



Perfect Binary Tree

- A full binary tree in which all leaf nodes are at the same level.



Complete Binary Tree

- A binary tree of height **h**, is a perfect binary tree down to height **h – 1** and the nodes at the lowest level are filled