Overview + Introduction

- Give handouts + introduce.
  - show course webpage
  - weekly HW assignments - due on Mondays.
  - office hrs - flexible. except 3:30-4:30 Mondays.
  - two class exams Mon 26 Sept, Mon 7 Nov.
  - Final exam TBA.

- Text: - note the texts on reserve.

- What is quantum mechanics?
  - General theory to describe workings of all physical systems. Fundamental part of physics.
  - Quantum mechanics has characteristic features that are very different from ordinary mechanics. These are most readily evident in
    - particle physics
    - physics of atoms + molecules.
    - physics of light
    - condensed matter physics
    - quantum information.

- What will this course cover?
  - A small portion of the body of knowledge of QM!
  - Lay out the basic framework so that you could adapt it to various other disciplines.
  - Much of the key ideas will be illustrate using spin-1/2 particles but we will also cover harmonic oscillator + hydrogen atom.
- What is the subject like?
  - abstract mathematical formulation.
  - useful mathematics — differential/integral calculus
  - differential equations
  - linear algebra.

Section 1  Review of Wave Mechanics

Wave mechanics typically deals with particles which move in one or more dimensions. How do we describe these?

A. Classical mechanics.

- Want the position + momentum of a particle as time passes.
- Show slide of general classical mechanics set-up
- Show slide of harmonic oscillator.

- Given the trajectory of a classical particle, \( x(t) \), we can derive everything else from this.

  velocity: \( v(t) = \frac{dx}{dt} \)

  acceleration: \( a(t) = \frac{d^2x}{dt^2} \)

  momentum: \( p(t) = m\frac{dx}{dt} \)

  energy: \( E = \frac{1}{2} m v^2 + V(x) \).

  etc....

- In summary, the main goal in classical mechanics is to determine the trajectory of the particles; everything else can be derived from this.
- Consider a particle which moves in just one dimension, we can denote the position variable by $x$.

- In wave mechanics, we found ourselves forced to describe the behavior of the particle in terms of waves. One of the motivations for this are the results of particle diffraction experiments.

- Show Hitachi website.

- Eventually we settled on the following description:

  i) The state of the system is described by a complex-valued wavefunction $\Psi(x,t)$, which must satisfy the Schrödinger equation:

  \[
  i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V(x) \Psi
  \]

  where $m =$ mass of particle

  \[
  \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} \quad \text{(Planck's constant)}
  \]

  $V(x) =$ potential describing external influences on particle.
Example: Infinite Square Well:

\[ V(x) = \begin{cases} 
0 & 0 \leq x \leq L \\
\infty & \text{otherwise} 
\end{cases} \]

Some solutions to Schrödinger's equation are the energy eigenstates:

\[ \Psi_n(x,t) = A \sin \left( \frac{n\pi x}{L} \right) e^{-iE_n t/\hbar} \]

where

\[ E_n = n^2 \frac{\pi^2 \hbar^2}{2mL} \]

- Show these for Falstad animation.

ii) The wavefunction must be normalized:

\[ \int_{-\infty}^{\infty} |\Psi_n(x,t)|^2 \, dx = 1. \]

Example: Infinite square well - this results in \( A = \sqrt{\frac{2}{L}} \). So

\[ \Psi_n(x,t) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi x}{L} \right) e^{-iE_n t/\hbar} \]
The wavefunction will be used to make statistical predictions about the outcomes of measurements on the particles. The wavefunction is best suited for describing position measurements. The main rule is

\[
\text{Position measurement gives an outcome in region } a \leq x \leq b \text{ with probability:}
\]

\[
\text{Prob}(a \leq x \leq b) = \int_a^b |\Psi(x, t)|^2 \, dx
\]

and we frequently find it useful to work with the probability density

\[
P(x, t) = |\Psi(x, t)|^2
\]

Show Falstad demo to give \( P(x, t) \) for various energy states, - ask about probabilities for various regions.