Overview of Condensed Matter Physics

Condensed matter physics deals with the properties of bulk materials such as:
- metals
- crystalline materials, e.g. salt NaCl
- rubber
- glass

The general aim is to describe measurable properties of these starting with fundamental physical principles. For example, what is the conductivity of copper and how does it vary with temperature?

One answers this type of question by realizing that conductivity has to do with transport of electrons through the copper. So it will require some concepts from E/M. But electrons will interact with each other and with the copper nuclei, and this will clearly require an understanding of the quantum mechanics that describes their interactions. Finally, we are seldom interested in single electrons flowing through the copper, but rather a sizeable number of electrons. So you will want some notion of the average behavior and this requires statistical mechanics.

So you can see that condensed matter physics brings together major fundamental branches of physics and will enable us to answer practical questions.
Traditionally condensed matter physics has relied heavily on the notion that bulk materials are built from atoms arranged in a regular symmetric fashion. Such structures are called crystals and their regularity vastly simplifies the study of bulk materials. Much of this course will focus on such crystalline materials. Briefly we will discuss:

- crystal structure + diffraction/scattering by crystals.
- vibrations of crystals → thermal properties of materials.
- electrons in crystalline structures → electrical properties.
- various special topics.

The subject is vast - the biggest currently in physics - and we will barely touch on it's range of topics.

Course structure:

Weekly HW → 40%
Two exams → 30% dates!
Term paper + presentation → 30% dates!

I will assign reading before each class. For Friday read: Omar p 2-612. 16-7 20.
I. Crystal Structures

We will start by considering a material consisting of one atomic species, for example pure Aluminium. In the solid state how are these atoms arranged? It may seem, since Aluminium comes in many shapes, that the individual atoms are arranged haphazardly. But we will see that this is not the case and that there is an orderly repeatable pattern to much of any piece of Aluminium.

Let's illustrate this with a simple demonstration involving solid spheres in a container.

Demonstration - Empty the container
- Pour some of the spheres in until you have about one layer that is half full. Observe the arrangement of the spheres. Is there any pattern?
- Now pour more spheres in until you have almost one layer - you need some wiggle room. Shake the container until patterns begin to emerge. What do you see?
- Try adding more to get a second layer.

With any luck you may observe that the spheres arrange themselves in a regular way for at least part of the container. You may notice a square arrangement:

![Square Arrangement Diagram]

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Or perhaps you noticed a hexagonal structure:

There may even have been a combination of these structures. Or you may have noticed two of the same structure offset by some sort of dislocation.

For both of the illustrated structures, you will notice that there is a regular pattern. We will begin by considering such structures that extend infinitely far in all directions. Such idealizations are called crystal structures.

The first part of this course is devoted to exploring these. We have two broad aims:

1) Develop a mathematical language for describing these
2) Describe how you can demonstrate that such structures exist for real materials and how you can determine which type of crystal structure a given material has.

1. Mathematical Description of Two Dimensional Crystal Structures.

Consider the square structure. We don't need to illustrate the actual spheres - rather we can just describe the location of the center of each sphere. The collection of these centers is called a crystal lattice.
We can arrange these on a grid

Note that the grid is very regular and has a high degree of symmetry. Let's consider these. The regularity is apparent from the fact that the grid can be constructed from a basic "tile" or cell. A shaded cell is indicated in the sketch. The construction proceeds as:

1) Translate a copy of the cell by the vector \( \vec{a}_1 \) as illustrated. This produces cell \( a_1 \).
2) Translate a copy of cell \( a_1 \) by the vector \( \vec{a}_2 \) as illustrated. This produces cell \( a_2 \).
3) Starting from each of cells \( a_1 \) or \( a_2 \) repeat the previous steps.
4) Starting from cell \( a_0 \) repeat steps 1-03 using the vectors \(-\vec{a}_1\) and \(-\vec{a}_2\).

Repeating this process, we can see that we generate an infinite array of lattice points. This is called a crystal structure.

Exercise: Do the same for a hexagonal structure in which nearest neighboring lattice points are a distance \( a \) apart. Describe the vectors which you used.
We have seen that regular structures can be created by repeated translations of a basic cell. If we imagine all the translations starting at one cell, for example $0$ in the square lattice, you should be able to convince yourself that these are described by the translation vectors:

$$\vec{R} = n_1 \vec{a}_1 + n_2 \vec{a}_2$$

where $n_1, n_2$ are integers. The cells and the translation vectors are in a one-to-one correspondence. For example, in the diagram:

<table>
<thead>
<tr>
<th>Cell</th>
<th>Vector</th>
<th>$n_1$</th>
<th>$n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$\vec{a}_1$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$\vec{a}_2$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>$3\vec{a}_1 + \vec{a}_2$</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

You should be convinced that the crystal lattice can be described by specifying the set of translation vectors and the contents of each cell, i.e. an atom at each corner, shared amongst four adjacent cells. But now notice that you really do not need to list all translation vectors - you merely need to describe:

i) $\vec{a}_1, \vec{a}_2$ and

ii) the contents of each cell.
We can see how this generates a lattice starting at just one point. Starting at A, one can translate by \( \hat{a}_1 \) to reach B or by \( \hat{a}_2 \) to reach D. Now repeat this process starting at the new points (B and D). Also repeat using \( -\hat{a}_1 \) and \( -\hat{a}_2 \). This clearly generates the whole lattice.

This can often be clearer than the "tiling" process involving cells, since one does not have to be concerned about the contents of the cells.

So we can adopt as the definition of a crystal lattice:

The set of all points whose position vectors (starting from a common origin) are:

\[ n_1 \hat{a}_1 + n_2 \hat{a}_2 \]

where \( \hat{a}_1 \) and \( \hat{a}_2 \) are two non-parallel vectors and \( n_1, n_2 \) integers.

Thus to specify a two-dimensional lattice one only needs two vectors \( \hat{a}_1, \hat{a}_2 \). Each point generated above is called a lattice site.

**Examples:**

1) Square lattice:

\[
\hat{a}_1 = a\hat{x} \\
\hat{a}_2 = a\hat{y}
\]

This clearly generates the lattice in the diagram.

2) Square lattice:

\[
\hat{a}_1 = a\hat{x} \\
\hat{a}_2 = \frac{1}{\sqrt{2}} (a\hat{x} + a\hat{y})
\]

This also generates a square lattice.
Example: The scheme for generating such crystal structures will eliminate some seemingly regular structures. One example is illustrated below:

Convince yourself that there are no vectors $\hat{a}_1$ and $\hat{a}_2$ which can generate all lattice points, starting at A.

Exercise: Produce a two dimensional lattice which is neither square nor hexagonal.

Exercise: Produce a two dimensional structure which is highly regular but is not a lattice. Hint: consider hexagons with no central lattice site.

Crystal Symmetries.

One consequence of the lattice construction involving basic vectors is that the entire lattice is invariant under any translation $\mathbf{R} = n_1 \hat{a}_1 + n_2 \hat{a}_2$ where $n_1$ and $n_2$ are integers. This means that if every lattice site is subjected to the same operation, then after the operation has been done, the appearance of the lattice has not changed - it has not even shifted.
We say that the lattice is symmetrical under the group of all translations generated by \( \vec{a}_1, \vec{a}_2 \).

Every lattice has this kind of translational symmetry. However, some lattices clearly have additional symmetries. That is, they are invariant under other geometrical operations. These fall into two classes:

1) rotations about one fixed point

2) reflections about a line.

Exercise: Describe additional symmetries that the square and hexagonal lattices have. Can you use these to distinguish between the lattices.

One can classify all lattices by considering such symmetries. It turns out that in two dimensions, there are only 5 distinct types of lattice.