ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Policies and Review Topics for Exam #1

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89.
2. You will be allowed to use one 8.5 × 11-inch two-sided handwritten help sheet. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheet that I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you later.
4. **You may not leave the exam room without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible.

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for the exam.

Electromagnetic spectrum
- allocation of frequencies is determined by international agreement
- choice of frequency for radio service can be dictated by:
  - antenna size (which in turn depends on wavelength)
  - absorption/reflection properties of atmosphere, buildings, foliage, etc.
  - ionospheric refraction
  - other propagation conditions
  - capabilities of current technology
  - government regulations and/or treaty obligations

Permittivity of free space (ε_0) = 8.854×10^{-12} F/m

Permeability of free space (μ_0) = $4\pi \times 10^{-7}$ H/m

Phasors
- complex numbers that represent sinusoidal functions in time
- one standard definition (the one we use): $A(t) = \mathrm{Re}\{\tilde{A}e^{j\omega t}\}$
- in the textbook, phasors are represented by the tilde symbol (~) over the variable
- all phasors are generally complex, but not all complex numbers are phasors (e.g., impedance is not a phasor quantity)
- if $\tilde{A} = |\tilde{A}|\, e^{j\phi}$, then $A(t) = |\tilde{A}|\cos(\omega t + \phi)$; A and ϕ can be functions of location
- phasors can be functions of any variable (usually spatial variables such as x, y, and/or z) **except** time

- $\dfrac{\partial A(t)}{\partial t} \leftrightarrow j\omega\tilde{A}$ and $\dfrac{\partial^2 A(t)}{\partial t^2} \leftrightarrow (j\omega)^2\tilde{A} = -\omega^2\tilde{A}$
- addition and subtraction in time domain are equivalent to addition and subtraction in phasor domain
- multiplication and division in time domain are **not** equivalent to multiplication and division in phasor domain

Complex arithmetic

- addition and subtraction; multiplication and division
- complex conjugate (*)
- polar form vs. rectangular form, and conversion between forms
- different types of polar form: e.g., $2\angle 30° = 2e^{j\pi/6}$
- how to compute the values of complex exponentials (like $e^{j\pi/6}$) using a calculator; must use radians with most calculator models; check degrees vs. radians setting
- identification of magnitude and phase
- identification of real part and imaginary part
- graphical representation of complex nos. on the complex plane (Im part vs. Re part)
- Euler's identity: $e^{j\theta} = \cos\theta + j\sin\theta$
- $|z| = \sqrt[+]{zz^*}$ (z is a complex number; superscript * represents conjugation)
- $|e^{j\theta}| = 1$, regardless of the value of θ, where θ = a real-valued constant or function
- $j = e^{j(0.5\pi \pm 2\pi n)}$, $-j = e^{j(-0.5\pi \pm 2\pi n)}$, and $-1 = e^{\pm j(\pi \pm 2\pi n)}$; where $n = 0, 1, 2, \ldots$

Transmission line fundamentals

- What constitutes a TEM line? (two long parallel conductors separated by dielectric; there must be two conductors across which a voltage can be measured; current must be able to flow in one direction along one conductor and back via the other conductor)
- xmsn line analysis necessary: line length is a significant fraction of λ or longer
- xmsn line analysis not necessary: line length is a negligible fraction of λ
- electrical length (in degrees, radians, or wavelengths) vs. physical length (in mm, cm, m, etc.) of transmission lines
- characteristic impedance Z_0 is the ratio of v to i for forward-propagating wave and also for reflected wave; Z_0 is dependent on geometry and material properties
- concepts of "source" (or "generator") and "load"
- concepts of "lumped elements" and "distributed" quantities (R', L', G', and C')
- lumped-element model ($R'\Delta z$, $L'\Delta z$, $G'\Delta z$, and $C'\Delta z$) – but you do not need to memorize the formulas in Table 2-1
- L' and C' are directly proportional to μ and ε, respectively, regardless of line type
- common types of transmission lines (basic geometries; see Fig. 2-4 in Ulaby & Ravaioli)
    - coaxial cable
    - two-wire line
    - microstrip line
    - parallel-plate line
- transmission line (or telegrapher's) equations:

  time domain: $-\dfrac{\partial v(z,t)}{\partial z} = R'i(z,t) + L'\dfrac{\partial i(z,t)}{\partial z}$ and $-\dfrac{\partial i(z,t)}{\partial z} = G'v(z,t) + C'\dfrac{\partial v(z,t)}{\partial z}$

  phasor domain: $-\dfrac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z)$ and $-\dfrac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z)$

- wave equation for phasor voltage and/or current along transmission lines
  $$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0 \quad \text{and} \quad \frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$
- complex propagation constant: $\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$
- solutions to the phasor wave equation for both voltage and current:
  - $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} = V_0^+ \left(e^{-\gamma z} + \Gamma e^{\gamma z}\right)$
  - $\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} = \dfrac{V_0^+}{Z_o}\left(e^{-\gamma z} - \Gamma e^{\gamma z}\right)$
- meanings of V_0^+ and V_0^- (and of I_0^+ and I_0^-); all four quantities can be complex
- how to find voltage (current) in time domain from phasor representation & vice versa
- cosine representation of waves in time domain: $v(z, t) = Ae^{-\alpha z}\cos(\omega t - \beta x + \phi)$

  A = amplitude

  α = attenuation constant (in Np/m; zero for lossless lines)

  ω = radian frequency (in rad/s)

  β = phase constant (in rad/m)

  ϕ = constant phase shift (comes from complex coef. like V_0^+)
- phasor representation of waves in frequency domain
- linearity and superposition (two or more waves traveling over the same transmission line simply add together)
- characteristic impedance (Z_0)
  - how it is derived (where it comes from); what it represents
  - dependence on xmsn line's geometry and material properties
  - $I_0^+ = \dfrac{V_0^+}{Z_0}$ and $I_0^- = -\dfrac{V_0^-}{Z_0}$
- meanings of real and imaginary parts of propagation constant: $\gamma = \alpha + j\beta$
  - α = attenuation constant, measured in Np/m or dB/m

    α [dB/m] = (20 log e)α [Np/m] = 8.68α [Np/m]
  - β = phase constant (or wavenumber), measured in rad/m
- permeability of all parts of almost all xmsn lines is free space value (i.e., $\mu = \mu_0$)
  - typically, $\mu \neq \mu_0$ only for iron, cobalt, nickel, and their compounds/alloys
  - "nonmagnetic" means $\mu = \mu_0$
- definitions of and relationships between:
  - T = period (measured in s)
  - λ = wavelength (m)
  - f = frequency (Hz, or 1/s)
  - ω = radian frequency (rad/s)
  - u_p = phase velocity (m/s)
  - ϕ = phase (rad or deg)
- phase velocity of waves along transmission lines:
  - derivation using $\dfrac{\partial}{\partial t}\cos(\omega t - \beta z) = 0$, or using

    $\cos(\omega t - \beta z) = \cos[\omega(t + \Delta t) - \beta(z + \Delta z)] =$ constant, where Δz is the distance traveled in time Δt by any single point on the cosine wave

  - $u_p = \omega/\beta$ (i.e., u_p is positive and wave travels in $+z$ direction) if one of the signs before ωt and βz is positive and the other is negative
  - $u_p = -\omega/\beta$ (i.e., u_p is negative and wave travels in $-z$ direction) if signs before ωt and βz are either both positive or both negative
  - $u_p = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{c}{\sqrt{\varepsilon_r}} = (\text{v.f.})c$, where v.f. = velocity factor (v.f. = $\varepsilon_r^{-1/2}$)
- for a lossless transmission line ($R' = 0$ and $G' = 0$),

  $$\alpha = 0, \quad \beta = \omega\sqrt{L'C'} = \frac{\omega\sqrt{\varepsilon_r}}{c}, \quad Z_0 = \sqrt{\frac{L'}{C'}}, \quad L' = \frac{Z_0}{u_p}, \quad \text{and} \quad C' = \frac{1}{Z_0 u_p}$$
- for a low-loss non-magnetic transmission line ($R' << \omega L'$ and $G' << \omega C'$),

  $$\alpha \approx \frac{R'}{2Z_0} + \frac{G'Z_0}{2}, \ \ \beta \approx \omega\sqrt{L'C'} = \frac{\omega\sqrt{\varepsilon_r}}{c}, \ \ Z_0 \approx \sqrt{\frac{L'}{C'}}, \ \ L' \approx \frac{Z_0}{u_p}, \ \ \text{and} \ \ C' \approx \frac{1}{Z_0 u_p},$$

  where Z_0 is an almost purely real value
- units of sinusoidal quantity vs. units of cos/sin argument (The magnitude of the cosine function represents a voltage or current and is measured in V or A, but the argument is measured in radians. That is, both ωt and βz are in radians.)

Loaded transmission lines

- In general, $\dfrac{\tilde{V}_{tot}(z_1)}{\tilde{I}_{tot}(z_1)} \neq \dfrac{\tilde{V}_{tot}(z_2)}{\tilde{I}_{tot}(z_2)}$ where z_1 and z_2 are two different locations along line
- Ohm's law must be satisfied at the load; that is, $Z_L = \dfrac{\tilde{V}_{tot}(0)}{\tilde{I}_{tot}(0)} = \dfrac{\tilde{V}_L}{\tilde{I}_L}$
- voltage reflection coefficient (Γ)
  - complex-valued (be able to find magnitude $|\Gamma|$ and phase θ_r): $\Gamma = |\Gamma| e^{j\theta_r}$
  - definition: $\Gamma = \dfrac{V_0^-}{V_0^+}$ usual formula: $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$
  - special cases:
    - if $Z_L = 0$, then $\Gamma = -1$
    - if $Z_L \to \infty$, then $\Gamma = +1$
    - if $Z_L = Z_0$, then $\Gamma = 0$
    - if Z_L is purely reactive (imaginary), then $|\Gamma| = 1$
  - phase of reflection coefficient usually expressed in the range $-\pi \leq \theta_r \leq \pi$
- expressions for $|\tilde{V}(z)|$ and $|\tilde{I}(z)|$ and their derivations

  $$|\tilde{V}(z)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|\cos(\theta_r + 2\beta z)\right]^{1/2}$$

  $$|\tilde{I}(z)| = \frac{|V_0^+|}{Z_0} \left[1 + |\Gamma|^2 - 2|\Gamma|\cos(\theta_r + 2\beta z)\right]^{1/2}$$
- interpretation of plots of $|\tilde{V}(z)|$ and $|\tilde{I}(z)|$ as standing waves
- voltage magnitude maxima and minima ($V_{\max}$ and $V_{\min}$)
- current magnitude maxima and minima ($I_{\max}$ and $I_{\min}$)
- voltage maxima (current minima) occur where $\cos(\theta_r + 2\beta z) = +1$
- voltage minima (current maxima) occur where $\cos(\theta_r + 2\beta z) = -1$

- voltage maxima and minima separated by $\lambda/4$
- current maxima and minima separated by $\lambda/4$
- voltage and current maxima and minima repeat every $\lambda/2$
- voltage maxima and current maxima separated by $\lambda/4$
- For the coordinate system we are using, physically meaningful values of z are negative, and physically meaningful values of d or l (distance to load or length of line) are positive.
- voltage standing wave ratio: $\text{VSWR} = \dfrac{1+|\Gamma|}{1-|\Gamma|} = \dfrac{V_{\max}}{V_{\min}}$ (for lossless lines)
    - VSWR = 1 and $V_{\min} = V_{\max}$ on a perfectly matched line ($Z_L = Z_0$)
    - VSWR $\rightarrow \infty$ and $V_{\min} = 0$ if load is 0, ∞, or purely imaginary
    - VSWR is often easier and/or cheaper to measure than Γ
    - VSWR can be the same for many different load impedances (because VSWR is a function only of $|\Gamma|$, not of the full complex quantity)

<u>Relevant course material:</u>

HW: #1 and #2
Reading: Assignments from Jan. 17 through Jan. 29

This exam will focus primarily on the course outcomes listed below and related topics:

1. Predict voltages, currents, and/or power flow along a transmission line given the line parameters and the signal source and load connected to the line.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.