ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Policies and Review Topics for Exam #3

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89.
2. You will be allowed to use up to three 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheet that I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you later.
4. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
5. **You may not leave the exam room without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheets for the previous exams as well.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for the exam.

Power flow along transmission lines

- be able to calculate power delivered by voltage/current source (generator), absorbed in source resistance (or impedance), delivered to input of line, and/or absorbed by load
- be able to calculate incident power P_{inc} and reflected power P_{ref}
- available power (P_A) = maximum power that can be delivered by source; this occurs when the load on the source (which might not equal the load on a transmission line) equals the complex conjugate of the Thévenin equivalent source impedance (Z_g)
- $P_{inc} = P_A$ if $Z_0 = Z_g$ (char. impedance = generator impedance)
- incident (P_{inc}) vs. reflected (P_{ref}) power and their relationship to power delivered to load

  $$P_L = P_{inc} - P_{ref} = \frac{|V_0^+|^2}{2Z_0}\left(1-|\Gamma|^2\right)$$

  Note that the textbook uses $P_L = P_{inc} + P_{ref}$ but defines P_{ref} as a negative value to indicate that it is "delivered" by the load back to the transmission line. Either interpretation (this one or the textbook's) is fine; just pay attention to context.
- It is acceptable to represent P_{ref} as a positive quantity if $P_L = P_{inc} - P_{ref}$ is used.
- conservation of power applies everywhere at all times

- calculation of time-average real power from phasor quantities:
  [Note that the textbook uses P_{av} to represent P_L (power delivered to load), whereas P_{av} is used here to denote time-average power in the general sense.]
  $$P_{av} = \frac{1}{2}\mathrm{Re}\{\tilde{V}\tilde{I}^*\} = \frac{1}{2}|\tilde{I}|^2 R = \frac{1}{2}|\tilde{V}|^2 \mathrm{Re}\left\{\frac{1}{Z^*}\right\} = \frac{1}{2}|\tilde{V}|^2 \mathrm{Re}\{Y^*\} = \frac{1}{2}|\tilde{V}|^2 G$$
  where V and I use peak amplitude units, not rms units (omit the "2" for the rms case)
  Note: $\mathrm{Re}\left\{\frac{1}{Z^*}\right\} = \mathrm{Re}\left\{\frac{1}{(R+jX)^*}\right\} = \mathrm{Re}\left\{\frac{1}{R-jX}\right\} \neq \frac{1}{R}$, if Z is complex (watch out!)
- net average real power delivered to load: $P_L = P_{inc}\left(1-|\Gamma|^2\right)$, where $P_{inc} = \frac{|V_0^+|^2}{2Z_0}$.
- return loss (in dB) = $-10 \log |\Gamma|^2 = -20 \log |\Gamma|$
  Note that return loss is a *positive* value in dB because $|\Gamma| < 1$. The corresponding reflection coefficient expressed in dB has a *negative* value, again because $|\Gamma| < 1$. The definition of return loss given here is an industry standard; however, some authors will carelessly express return loss as a negative value because they calculate it as $20 \log |\Gamma|$ (i.e., without the leading minus sign).
- return loss ≈ 10 dB, if $|\Gamma| = 0.333$ (VSWR = 2). This represents a case in which the reflected power is roughly 10% of the incident power. It is a widely accepted threshold for a "good" impedance match.

Antenna analysis (determination of radiation fields from current distributions)

- all time-varying currents act as sources and potentially can radiate EM waves. Examples:
  - antennas
  - the sun
  - lightning
  - sparks and other arc flashes
  - time-varying currents flowing in circuits (e.g., address bus in a computer, video signals, noise on power leads/bus)
  - charged particles accelerated by earth's magnetic field
- position vectors:
  - **R** (unprimed) defines observation point (distant point where E-field is calculated)
  - **R′** (primed) defines point on antenna
  - spherical coordinates: $\mathbf{R} = \hat{\mathbf{R}}R$, where direction of $\hat{\mathbf{R}}$ is a function of θ and ϕ
  - cylindrical coordinates: $\mathbf{R} = \hat{\mathbf{r}}r + \hat{\mathbf{z}}z$, where direction of $\hat{\mathbf{r}}$ is a function of ϕ
  - rectangular coordinates: $\mathbf{R} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z$

Exact electric and magnetic fields radiated by Hertzian dipole (infinitesimally short filament of current with "length" l, with uniform current distribution, located at origin, and aligned along z-axis; magnitude of input current $\tilde{I}_{in}$ is in peak units):

$$\tilde{\mathbf{E}} = \hat{\mathbf{R}}\frac{\tilde{I}_{in}k^2 l}{4\pi}\eta e^{-jkR}\left[\frac{2}{(kR)^2} - \frac{j2}{(kR)^3}\right]\cos\theta + \hat{\boldsymbol{\theta}}\frac{\tilde{I}_{in}k^2 l}{4\pi}\eta e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta$$

$$\tilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}}\frac{\tilde{I}_{in}k^2 l}{4\pi}e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2}\right]\sin\theta$$

Far field criterion ($kR >> 1$ for Hertzian dipoles)

Far fields of Hertzian dipole (uniform current distrib. in peak units) of length *l*:

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}} \frac{jk\eta \tilde{I}_{in} l}{4\pi R} e^{-jkR} \sin\theta, \quad \tilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}} \frac{jk\tilde{I}_{in} l}{4\pi R} e^{-jkR} \sin\theta$$

Common characteristics of *far*-field expressions (for any antenna orientation or location unless otherwise specified):

- the $\dfrac{e^{-jkR}}{R}$ factor, which implies spreading spherical waves (true for *all* antennas)
- propagation in $\hat{\mathbf{R}}$ direction (if antenna is centered at origin) (true for *all* antennas); the direction is directly away from the antenna
- speed of propagation is $\dfrac{1}{\sqrt{\mu\varepsilon}}$ (speed in surrounding medium; true for *all* transverse electromagnetic (TEM) waves)
- electric and magnetic fields are proportional to input current (true for *all* antennas fed by a transmission line)
- $\tilde{\mathbf{E}} \perp \tilde{\mathbf{H}}$, $\tilde{\mathbf{E}} \perp \mathbf{S}_{av}$, and $\tilde{\mathbf{H}} \perp \mathbf{S}_{av}$ (where $\mathbf{S}_{av}$ = time-average Poynting vector; true for *all* TEM waves; the "transverse" of TEM refers to orthogonality of **E** and **H** to **S**)
- electric and magnetic fields are in phase if η is purely real (true for *all* TEM waves)
- $\dfrac{|\tilde{\mathbf{E}}|}{|\tilde{\mathbf{H}}|} = \eta$ (true for *all* TEM waves)
- For Hertzian dipoles and all straight-wire antennas aligned along the *z*-axis, the far electric field is θ-directed, and the far magnetic field is ϕ-directed.

Time-average Poynting vector

- definition: $\mathbf{S}_{av} = \dfrac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$, if electric and magnetic fields are in peak units and expressed as phasors; alternate expression for TEM waves is

  $\mathbf{S}_{av} = \dfrac{|\tilde{\mathbf{E}}|^2}{2\eta}$ (no need to determine **H**)
- gives the power density per unit area of an EM wave (unit is the W/m^2)
- points in the direction of power flow and propagation direction of phase fronts (in most or all media that we are considering in this course)

Radiation pattern

- plot of $|\mathbf{S}_{av}|$ (sometimes normalized), directivity, or gain vs. θ and/or ϕ
- normalized power pattern:

  $F(\theta,\phi) = \dfrac{|\mathbf{S}_{av}|}{S_{\max}}$
- usually plotted using a dB (or dBi, for directivity and gain) scale
- interpretation of radiation pattern plot (either in terms of actual gain/directivity or the normalized power pattern)
- determination of relative power in various directions
- determination of half-power beamwidth

To find far fields of arbitrary current that flows along z-axis, use

$$d\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}} \frac{jk\eta I(z')dz'}{4\pi R} e^{-jkR'} \sin\theta \quad , \quad d\tilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}} \frac{jkI(z')dz'}{4\pi R} e^{-jkR'} \sin\theta$$

These are the far fields radiated by a Hertzian dipole of length dz, where primed coordinates refer to position z' along current distribution. Leads to

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}} \frac{jk\eta}{4\pi R} e^{-jkR} \sin\theta \int_{-l/2}^{l/2} I(z') e^{jkz'\cos\theta} dz'$$

Far fields of short dipole (triangular current distrib.) of length l (peak units):

$$\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}} \frac{jk\eta \tilde{I}_{in} l}{8\pi R} e^{-jkR} \sin\theta \,, \quad \tilde{\mathbf{H}} = \hat{\boldsymbol{\varphi}} \frac{jk\tilde{I}_{in} l}{8\pi R} e^{-jkR} \sin\theta$$

Approximation of current distribution along center-fed dipoles:

- start with current distribution on an open-circuited parallel-wire transmission line stub
- bend transmission line wires near end of stub outward 90° (fold point is $l/2$ from end of stub) so that folded wires are collinear (in line with each other)
- short dipole has a nearly triangular current distribution because the “ends” of the sinusoidal distribution are nearly linear
- Hertzian dipole has uniform current distribution, which is not physically possible, but it is a useful concept. Uniform current can be approximated using “capacity hats” (charge reservoirs) at the outer ends of the wire halves, which allow the currents at the ends of the dipole to be non-zero.

Directivity and gain

- calculation of radiated power (equal to input power if no losses)

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} \mathbf{S}_{av}(\theta,\phi)\cdot\hat{\mathbf{R}}R^2 \sin\theta \, d\theta \, d\phi = S_{\max} R^2 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta \, d\theta \, d\phi$$

- concept of isotropic radiator
  - o hypothetical antenna that radiates with equal intensity in all directions
  - o radiated fields have no specific polarization (not realistic)
  - o Poynting vector of isotropic radiator: $\mathbf{S}_{iso} = \hat{\mathbf{R}} \frac{P_{in}}{4\pi R^2}$, where P_{in} is input power to isotropic antenna, which is assumed to be lossless
- directivity calculated from power pattern

$$D = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\varphi)\sin\theta \, d\theta \, d\varphi}$$

- other relationships involving directivity

$$D = \frac{S_{\max}}{|\mathbf{S}_{iso}|} = 4\pi R^2 \frac{S_{\max}}{P_{rad}} = 4\pi R^2 \frac{S_{\max}}{P_{in}}$$ (2nd equality assumes **no power losses**)

- if there are power losses, then $G = \xi D$, where G is the gain and ξ is the efficiency. Also,

$$G = \frac{S_{\max}}{|\mathbf{S}_{iso}|} = 4\pi R^2 \frac{S_{\max}}{P_{in}} = 4\pi R^2 \xi \frac{S_{\max}}{P_{rad}}$$, because $P_{rad} = \xi P_{in} \quad \rightarrow \quad P_{in} = \frac{P_{rad}}{\xi}$

- gain & directivity are usually expressed in dBi (decibels relative to an isotropic radiator):
  D[dBi] = 10 log(D) $D = 10^{D[\text{dBi}]/10} = 10^{0.1D[\text{dBi}]}$
  G[dBi] = 10 log(G) $G = 10^{G[\text{dBi}]/10} = 10^{0.1G[\text{dBi}]}$
  Also note that
  $G = \xi D \;\rightarrow\; 10\log(G) = 10\log(\xi) + 10\log(D) \;\rightarrow\; G[\text{dBi}] = \xi[\text{dB}] + D[\text{dBi}]$
  where ξ, because it is less than 1, always has a negative value in dB
- dBi unit (gain/directivity in dB referenced to isotropic radiator) vs. dB unit
- directivities of short dipole, Hertzian dipole, and small loop are all 1.5 (1.76 dBi) because normalized power patterns are all $\sin^2\theta$

Radiation resistance

- input impedance of antenna: $Z_{in} = R_{rad} + R_{loss} + jX_{in}$,
  where R_{rad} = radiation resistance, R_{loss} = loss resistance, X_{in} = input reactance
- R_{rad} is real part of equivalent input impedance that represents radiated power; it accounts for power delivered by transmission line that is radiated by antenna
- definition: $R_{rad} = \dfrac{2P_{rad}}{|I_{in}|^2}$, if I_{in} represents peak (not rms) input current at the feed point;
  however, I_{in} might not be the peak value of the current distribution along the antenna
- short dipole: $R_{rad} = 20\pi^2\left(\dfrac{\Delta z}{\lambda}\right)^2$
- Hertzian dipole: $R_{rad} = 80\pi^2\left(\dfrac{\Delta z}{\lambda}\right)^2$
- half-wave dipole: $R_{rad} = 73\ \Omega$ (ideal half-wave dipole isolated in free space)
- quarter-wave monopole: $R_{rad} = 36.5\ \Omega = 73/2\ \Omega$ (ideal monopole over perfectly conducting ground plane of infinite extent)

Gain (G) and efficiency (ξ)

- $G = \xi D$, where D is the directivity
- $P_{rad} = \xi P_{in}$
- $\xi = \dfrac{R_{rad}}{R_{rad} + R_{loss}}$, if R_{rad} and R_{loss} are in series in the input impedance model
- loss resistance usually represents finite conductivity of antenna structure and/or ground beneath it, but other factors can contribute to loss as well
- calculation of power density at a distance given gain or directivity and input power to antenna (does not include xmsn line losses):
  $S_{\max} = \dfrac{P_{in}G}{4\pi R^2} = \dfrac{P_{in}\xi D}{4\pi R^2}$

Relevant course material:

HW: #5 and #6
Reading: Assignments from Feb. 19 through Mar. 18, including
“Radiation Power and Directivity of Antennas”
“Radiation Resistance, Efficiency, and Gain of Antennas”

This exam will focus primarily on the course outcomes listed below and related topics:

1. Predict voltages, currents, and/or power flow along a transmission line given the line parameters and the signal source and load connected to the line. [focus on power]
3. Relate the power density of an electromagnetic wave radiated in any direction to an antenna’s gain, radiation pattern, and applied input power.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.