ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Policies and Review Topics for Exam #4

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-89. Please make sure that you know how to use all of the relevant features of your calculator, especially those related to complex numbers and the representation of phase. Lack of proficiency with your calculator that leads to incorrect solutions will not be considered an extenuating circumstance. If you do not know how to use a particular feature of your calculator, then you must complete the calculations in question manually. Assistance with the operation of your calculator cannot be provided during the exam.
2. You will be allowed to use up to four 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheet that I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you later.
4. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
5. **You may not leave the exam room before completing your exam without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.**

The exam will take place 1:00–2:00 pm on Monday, April 15 in Academic West 116.

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheets for the previous exams as well.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate information when preparing for the exam.

Loss resistance R_{loss} due to finite conductivity of antenna

- real part of equivalent input impedance of antenna that accounts for power delivered by transmission line that is absorbed (not radiated) by antenna structure and perhaps by some surrounding objects and/or ground
- definition $R_{loss} = \frac{2P_{loss}}{|I_{in}|^2}$, with I_{in} in peak (not rms) units
- Hertzian dipole: $R_{loss} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$ (lower case “L” in numerator, not “1”)

short dipole: $R_{loss} = \frac{l}{6\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$

half-wave dipole: $R_{loss} = \frac{\lambda}{8\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$

arbitrary-length dipole: $R_{loss} = \frac{1}{2\pi a |I_{in}|^2}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}\int_{-l/2}^{l/2} |I(z)|^2 dz$

where l = length of wire; a = radius of wire (assumed constant along length); f = operating frequency; μ_c = permeability of wire (usually μ_0); σ_c = conductivity of wire

Specialized computational methods like the one used in EZNEC are required to find accurate current distributions along real antennas, but current distributions can be approximated by imagining the conductors of an open-circuited stub being folded outward $l/2$ from the end.

Half-wave dipole

- expression for far field if dipole is center fed, oriented along z-axis, and operating in free space:
  $$\tilde{\mathbf{E}} = \hat{\boldsymbol{\theta}}\, j60\tilde{I}_{in}\frac{e^{-jkR}}{R}\left[\frac{\cos(0.5\pi\cos\theta)}{\sin\theta}\right]$$
- directivity is 1.64 (2.15 dBi)
- radiation resistance is approx. 73 Ω
- input reactance behaves much like that of an open-circuited transmission line stub in the neighborhood of $\lambda/4$ in length; X_{in} is theoretically zero for exact $\lambda/2$ length of dipole
- current distrib. on a real half-wave dipole is not exactly sinusoidal, so actual resonant length is slightly less than $\lambda/2$

Faraday's law

- time domain form: $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$; phasor form: $\nabla \times \tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{B}}$
- integrate over open surface and apply Stokes' theorem to obtain integral forms (time-domain and phasor-domain cases shown below):
  $V_{emf} = \oint_C \mathbf{E}\cdot d\mathbf{l} = -N\iint_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}$ and $V_{emf} = \oint_C \tilde{\mathbf{E}}\cdot d\mathbf{l} = -j\omega N\iint_S \tilde{\mathbf{B}}\cdot d\mathbf{s}$; N = no. of turns
- sum of voltage drops around contour C is equal to the surface integral of time rate of change of magnetic flux density *crossing* surface
- right-hand rule relates direction of contour C (and therefore the direction of integration of the contour integral) to surface normal $\hat{\mathbf{n}}$
- polarity of V_{emf}: direction of C points from + to − at the terminals; with that polarity, V_{emf} has a *negative* value if **B** points in direction of surface normal and is increasing.
- A loop with voltage and current induced in it acts as a source to any external circuit that might be connected to it.

Lenz's law

- direction of actual current induced in loop is such that the **B** field it creates opposes the change in the external **B** field that produced the current
- satisfies law of conservation of energy

Small loop antennas (circumference << λ)

- often used in portable devices operating at "low" frequencies ~VHF/UHF and below
- **B** field must have a nonzero component that is normal to the loop in order to induce voltage at terminals
- radiation pattern null in the directions normal to plane of loop
- for a small transmitting loop of area *A* in *xy*-plane and centered at origin, far field is $\tilde{\mathbf{E}} = \hat{\boldsymbol{\varphi}} \frac{\eta k^2 A \tilde{I}}{4\pi} \frac{e^{-jkR}}{R} \sin\theta$, where $\tilde{I}$ is peak (not rms) value of input current; *A* = loop area; η = intrinsic impedance of surrounding medium; $k = 2\pi/\lambda$. Loop area can be any shape, but circular area maximizes area for a given conductor length (greatest efficiency).
- radiation resistance of *N*-turn loop of area *A* in free space

  $$R_{rad} = 31{,}200 N^2 \left(\frac{A}{\lambda^2}\right)^2$$
- radiation resistance of an *N*-turn circular loop of radius *r* in free space

  $$R_{rad} = 308{,}000 N^2 \left(\frac{r}{\lambda}\right)^4$$
- loss resistance of loop with total wire length *l* and wire radius *a* is

  $R_{loss} = R_{ohmic} = \frac{l}{2\pi a} R_s$, where $R_s = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$ ("*l*" in numerator, not "1")
- inductive reactance of loop is usually tuned out by addition of a capacitor
- electrically large loops (i.e., > 0.05λ to 0.1λ or so in diameter) have very different radiation patterns compared to those of small loops; significant radiation normal to loop; 1λ circumference loop has a radiation peak normal to the loop

Effective aperture (or effective area)

- relationship to gain or directivity

  $A_e = \frac{\lambda^2 D}{4\pi} = \frac{\lambda^2}{4\pi}\frac{G}{\xi}$ or $D = \frac{4\pi}{\lambda^2} A_e$ or $G = \xi \frac{4\pi}{\lambda^2} A_e$, where ξ = power efficiency
- effective aperture related to physical aperture size (area) A_{ap} by aperture efficiency (ξ_{ap}); ξ_{ap} is not the same as ξ

  $$A_e = \xi_{ap} A_{ap}$$
- beware of mixing multiplying factors and values in dBi in formulas for *D* and *G*; also remember to convert ξ expressed in % to fractional value; e.g., if *G* = 3 dBi, substitute 2, not 3, for *G* in formula.

Friis transmission formula

- used for "link budget" analysis
- assuming antennas are oriented in directions of max. radiation, are co-polarized, there are no reflections from nearby objects, there is no loss in the space between the antennas, and there are no losses along connecting transmission lines, then

  $$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi R}\right)^2 = P_{TX} \frac{\xi_{TX}\xi_{RX} A_{eTX} A_{eRX}}{\lambda^2 R^2},$$

  where TX refers to the transmitter and RX to the receiver

- if antenna(s) are not correctly oriented, then G (or D) for each antenna must be reduced by normalized power pattern value in direction of TX and/or RX:

$$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi R} \right)^2 F_{TX} \left(\theta_{TX}, \varphi_{TX} \right) F_{RX} \left(\theta_{RX}, \varphi_{RX} \right)$$

- transmission line loss:
    - reduces transmitted and/or received power
    - should not be included in efficiency; it is more properly treated separately because the transmission line is not part of the antenna
    - Loss has a negative dB value but is often expressed as a positive value in common usage; pay attention to context; negative dB value corresponds to loss factor < 1 (e.g., loss = 8 dB corresponds to a "gain" of −8 dB and a multiplying factor of $10^{-8/10} = 0.1585$)

<u>Relevant course material:</u>

HW: #7 and #8
Reading: Assignments from Mar. 21 through Apr. 8, including
"Radiation Resistance, Efficiency, and Gain of Antennas"
"Loss Resistance Calculations for Arbitrary Current Distributions"
"The Small Loop Antenna"

A supplemental sheet with Tables 2-1, 2-2, 2-4, and 7-1 from the textbook (Ulaby and Ravaioli, 7th ed.) plus several fundamental formulas will be made available to you during the exam. In addition, **four** 8.5 × 11-inch two-sided handwritten help sheets may be used during the exam.

This exam will focus primarily on the course outcomes listed below and related topics:

3. Relate the power density of a radiated electromagnetic wave to an antenna's gain, radiation pattern, and applied input power. [focus on loss resistance, gain, efficiency, the half-wave dipole, and effective aperture]
4. Perform link budget calculations using the Friis transmission formula.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.