ECEG 390 Theory and Applications of Electromagnetics Spring 2024

Final Exam General Information

The following policies will be in effect for the exam. They will be included in a list of instructions and policies on the first page of the exam:

1. You will be allowed to use a non-wireless enabled calculator, such as a TI-99. Please make sure that you know how to use all of the relevant features of your calculator, especially those related to complex numbers and the representation of phase. Lack of proficiency with your calculator that leads to incorrect solutions will not be considered an extenuating circumstance. If you do not know how to use a particular feature of your calculator, then you must complete the calculations in question manually. Assistance with the operation of your calculator cannot be provided during the exam.
2. You will be allowed to use up to **five** 8.5 × 11-inch two-sided handwritten help sheets. No photocopied material or copied and pasted text or images are allowed. If there is a table or image from the textbook or some other source that you feel would be helpful during the exam and that is not included on the table and formula sheet I will provide, please notify me.
3. All help sheets will be collected at the end of the exam but will be returned to you later if you wish to have them back.
4. If you begin the exam after the start time, you must complete it in the remaining allotted time. However, you may not take the exam if you arrive after the first student has completed it and left the room. The latter case is equivalent to missing the exam.
5. **You may not leave the exam room before completing your exam without prior permission except in an emergency or for an urgent medical condition. Please use the restroom before the exam.** If you have a medical condition that might require you to leave the room, you must notify me before the exam begins. Only one student at a time may be absent from the room and must leave any electronic devices in the room.

The final exam will take place **8:00–11:00 am on Monday, May 6 in Academic West 116**.

As stated in the ECEG 390 Course Policies & Information sheet (the syllabus), the final exam will consist of two parts. The first part will resemble the four in-semester exams and, like them, will be quantitative in nature. It will focus primarily on the material covered since Exam #4, but you will also need to have a good grasp of the material from earlier in the semester. The second part (the Final Concept Exam) will be mostly qualitative and will assess your understanding of the key course concepts. The two parts will be designed to take a total of approximately 1.5 hours to complete, but you may use the full three hours if necessary.

Your graded final exam will not be returned to you, nor will the solutions be posted. However, you may make an appointment with me at any time to review your final exam and discuss your performance on it. I will keep your final exam at least until you graduate from Bucknell.

Review Topics for Final Exam

The following is a list of topics that could appear in one form or another on the exam. Not all of these topics will be covered, and it is possible that an exam problem could cover a detail not specifically listed here. However, this list has been made as comprehensive as possible. **You should be familiar with the topics on the review sheets for all of the previous exams in addition to those listed below.**

Although significant effort has been made to ensure that there are no errors in this review sheet, some might nevertheless appear. The textbook is the final authority in all factual matters, unless errors have been specifically identified there. You are ultimately responsible for obtaining accurate and authoritative information when preparing for the exam.

Maxwell's equations in differential time-domain form (the "point-wise" equations):
- Gauss's law: $\nabla \cdot \mathbf{D} = \rho_v$
- Faraday's law: $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$
- "Magnetic Gauss's" law: $\nabla \cdot \mathbf{B} = 0$
- Ampére's law: $\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$

Maxwell's equations in differential time-harmonic (phasor) form:
- Gauss's law: $\nabla \cdot \tilde{\mathbf{D}} = \tilde{\rho}_v$
- Faraday's law: $\nabla \times \tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{B}}$
- "Magnetic Gauss's" law: $\nabla \cdot \tilde{\mathbf{B}} = 0$
- Ampére's law: $\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\tilde{\mathbf{D}}$

Constitutive relations (valid in time-domain form or time-harmonic, i.e., phasor, form), but all assume that the constitutive parameters are constant scalars (isotropic, nondispersive media):
- $\mathbf{D} = \varepsilon\mathbf{E}$, where ε is the permittivity of the medium
- $\mathbf{B} = \mu\mathbf{H}$, where μ is the permeability of the medium
- $\mathbf{J} = \sigma\mathbf{E}$, where σ is the conductivity of the medium

Source-free vs. source-filled regions (i.e., are $\mathbf{J}$ and/or ρ_v zero or non-zero?)
Can calculate electric (magnetic) field from magnetic (electric) field via:
- $\tilde{\mathbf{E}} = \dfrac{1}{j\omega\varepsilon}\nabla \times \tilde{\mathbf{H}}$ (source-free Ampére's law)
- $\tilde{\mathbf{H}} = \dfrac{-1}{j\omega\mu}\nabla \times \tilde{\mathbf{E}}$ (source-free Faraday's law)

Wave equations for time-harmonic fields in a source-free $\left(\tilde{\mathbf{J}} = 0,\ \tilde{\rho}_v = 0\right)$, lossless ($\varepsilon$ and μ are both real) region:
- $\nabla^2\tilde{\mathbf{E}} + k^2\tilde{\mathbf{E}} = 0$
- $\nabla^2\tilde{\mathbf{H}} + k^2\tilde{\mathbf{H}} = 0$
- $k^2 = \omega^2\mu\varepsilon$

Time domain forms of fields: $\mathbf{E}(t) = \mathrm{Re}\{\tilde{\mathbf{E}} e^{j\omega t}\}$ and $\mathbf{H}(t) = \mathrm{Re}\{\tilde{\mathbf{H}} e^{j\omega t}\}$, where tilde (~) over variable indicates that it is a phasor

Plane-wave propagation

- wave equations for time-harmonic fields in a source-free $\left(\tilde{\mathbf{J}} = 0,\ \tilde{\rho}_v = 0\right)$, lossy region
  - $\nabla^2 \tilde{\mathbf{E}} - \gamma^2 \tilde{\mathbf{E}} = 0$
  - $\nabla^2 \tilde{\mathbf{H}} - \gamma^2 \tilde{\mathbf{H}} = 0$
  - $\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu \left(\varepsilon' - j\varepsilon''\right)$
- time domain forms of fields: $\mathbf{E}(t) = \mathrm{Re}\{\tilde{\mathbf{E}} e^{j\omega t}\}$ and $\mathbf{H}(t) = \mathrm{Re}\{\tilde{\mathbf{H}} e^{j\omega t}\}$

Uniform plane waves

- "uniform:" there is no variation in field strength in directions normal (transverse) to prop. direction (e.g., if prop. is in z-direction, then $\partial/\partial x$ and $\partial/\partial y = 0$)
- "plane:" planar (flat) phase fronts, rather than spherical or cylindrical (or other shape)
- TEM (transverse electromagnetic) waves, planar or not:
  - **E** and **H** are both perpendicular to the dir. of prop. and are perpendicular to each other
  - neither **E** nor **H** has a component in the dir. of prop.
  - waves can be planar, cylindrical, spherical, or conforming to other kinds of nonstandard orthogonal coordinate systems, such as ellipsoidal
- for a TEM plane wave that has only an E_x component and is propagating in the $\pm z$-direction:
  - lossless medium: $\dfrac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0$ (similar eqn for H field)
  - lossy medium: $\dfrac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$ (similar eqn for H field)
- solutions:
  - lossless medium: $\tilde{\mathbf{E}} = \hat{\mathbf{x}} \tilde{E}_{xo} e^{\pm jkz}$ (similar eqn for H field)
  - lossy medium: $\tilde{\mathbf{E}} = \hat{\mathbf{x}} \tilde{E}_{xo} e^{\pm \alpha z} e^{\pm j\beta z}$ (similar eqn for H field)
- sign of jkz (or $j\beta z$) indicates direction of propagation
- for TEM waves, $\tilde{\mathbf{E}} = -\eta_c \hat{\mathbf{k}} \times \tilde{\mathbf{H}}$, and $\tilde{\mathbf{H}} = \dfrac{1}{\eta_c} \hat{\mathbf{k}} \times \tilde{\mathbf{E}}$; $\hat{\mathbf{k}}$ is unit vector in dir. of prop.; η_c is defined below
- can use Faraday's law to find **H** from **E** or source-free Ampère's law to find **E** from **H**
- $k = \beta = 2\pi/\lambda$, but pay attention to context
- general media (including quasi-conductors): neither $\dfrac{\varepsilon''}{\varepsilon'} << 1$ nor $\dfrac{\varepsilon''}{\varepsilon'} >> 1$
  - attenuation constant (in Np/m): $\alpha = \omega \left\{ \dfrac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} - 1 \right] \right\}^{1/2}$
  - phase constant (in rad/m): $\beta = \omega \left\{ \dfrac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2} + 1 \right] \right\}^{1/2}$

- $\eta_c = \sqrt{\dfrac{\mu}{\varepsilon' - j\varepsilon''}} = \sqrt{\dfrac{\mu}{\varepsilon'}}\left(1 - j\dfrac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$ (assumes no sig. magnetic loss; i.e., $\mu'' = 0$)

- low-loss dielectrics: $\dfrac{\varepsilon''}{\varepsilon'} << 1$ (assumes no significant magnetic loss; i.e., $\mu'' = 0$)
  - attenuation constant (in Np/m): $\alpha \approx \dfrac{\omega\varepsilon''}{2}\sqrt{\dfrac{\mu}{\varepsilon'}} = \dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon}}$
  - phase constant (in rad/m): $\beta \approx \omega\sqrt{\mu\varepsilon}$
  - $\eta_c \approx \sqrt{\dfrac{\mu}{\varepsilon}}$ (approximately real)
  - $\eta_o = 120\pi\ \Omega = 377\ \Omega$ in free space (vacuum or air)
- good conductors: $\dfrac{\varepsilon''}{\varepsilon'} >> 1$ (assumes no significant magnetic loss)
  - attenuation constant (in Np/m): $\alpha \approx \omega\sqrt{\dfrac{\mu\varepsilon''}{2}} = \sqrt{\dfrac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$
  - phase constant (in rad/m): $\beta \approx \sqrt{\pi f\mu\sigma}$
  - $\eta_c \approx (1+j)\sqrt{\dfrac{\pi f\mu}{\sigma}} = (1+j)\dfrac{\alpha}{\sigma}$ (assumes no significant magnetic loss)
- permeability is almost never complex, except in the case of lossy ferromagnetic materials such as iron, cobalt, nickel, and their alloys
- relationships between period T, frequency f (or ω), wavelength λ, and phase constant k (also called the wave number) of waves (or α and β)
- **E** and **H** are *in* phase if medium is lossless
- **E** and **H** are *out of* phase if medium is lossy (although never more than 45° for isotropic materials with constant ε, μ, and σ)
- speed of wave found from $\dfrac{\partial(\omega t - kz + \phi)}{\partial t} = 0$ or $\dfrac{\partial(\omega t - \beta z + \phi)}{\partial t}$; $u_p = \dfrac{\partial z}{\partial t}$, ϕ is const.
- perfect plane waves cannot exist in nature, but they are very good approximations of spherical waves (and other practical types of waves) over regions of limited size at large distances from wave sources

Polarization

- linear pol.
  - tip of E-field vector traces out a line segment over a full sinusoidal cycle (period)
  - both vector components (e.g., the x- and y-components of a wave traveling in z-direction) of E-field (and H-field) are in phase or 180° out of phase but might have unequal magnitudes (or one component could be zero)
  - two mutually perpendicular linear polarizations are possible. Implication: an antenna oriented to favor one polarization will be insensitive to the other.
- circular pol. (CP)
  - tip of E-field vector traces out a circle over a full cycle (period)
  - both vector components of E-field (or H-field) have equal magnitude but are ±90° out of phase
  - left-hand and right-hand circular polarizations are possible; antennas that have LHCP are not compatible with antennas that have RHCP, and vice versa

- o H-field vector rotates at same rate and in same direction as E-field vector (angular velocity is $\omega = 2\pi f$) and is always perpendicular to E-field vector
- elliptical pol. (EP)
  - o tip of E-field vector traces out an ellipse over a full cycle (period)
  - o both vector components of E-field (or H-field) are out of phase by an amount other than 0°, 180°, or ±90° (or out of phase by ±90° with unequal magnitudes)
  - o left-hand and right-hand elliptical polarizations are possible; LHEP might or might not be compatible with RHEP, and vice versa
  - o H-field vector rotates at same rate and in same direction as E-field vector and is always perpendicular to E-field vector
- Linearly polarized antennas can receive some of the power contained in a CP or EP wave.
- CP antennas can receive some of the power contained in a linearly polarized or EP wave.
- It is rare for antennas and other signal sources to be intentionally designed to produce elliptical polarization, but it is highly likely that a linearly polarized or CP wave will become elliptically polarized to some extent after reflection, refraction, diffraction, and other interactions with objects in a real environment.

Electromagnetic power density

- instantaneous Poynting vector (a time-dependent quantity): $\mathbf{S}(t) = \mathbf{E}(t) \times \mathbf{H}(t)$ (unit is W/m^2); represents the instantaneous power per unit area of a wave (general formula for any kind of time variation)
- time-average Poynting vector (a time-independent quantity) represents the time-average value of **S** and is applicable to time-harmonic (periodic) waves:

  $\mathbf{S}_{av} = \frac{1}{2}\mathrm{Re}\left\{\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^*\right\}$, where E and H are expressed in terms of their peak (not rms) values
- lossless media: $\mathbf{S}_{av} = \hat{\mathbf{k}}\dfrac{\left|\tilde{E}_1\right|^2 + \left|\tilde{E}_2\right|^2}{2\eta}$,where $\tilde{E}_1$ and $\tilde{E}_2$ are the two complex orthogonal vector components of the wave perpendicular to the direction of propagation and are expressed in terms of peak units.
- lossy media: $\mathbf{S}_{av} = \hat{\mathbf{k}}\dfrac{\left|\tilde{E}_1\right|^2 + \left|\tilde{E}_2\right|^2}{2|\eta_c|}e^{-2\alpha z}\cos\theta_\eta$, where $\eta_c = |\eta_c|e^{j\theta_\eta}$
- attenuation through the atmosphere increases substantially in millimeter wave range (20 GHz and above, primarily because of resonances of gas molecules)
- α[dB/m] = 8.68 α[Np/m]
- total power in wave intercepted by a surface of area A: $P_{av} = \iint_A \mathbf{S}_{av} \cdot d\mathbf{s}$

  If wave is incident normal to aperture and *uniform* over aperture: $P_{av} = |\mathbf{S}_{av}|A$

  If $\mathbf{S}_{av}$ is not uniform over aperture, then evaluation of integral is more complicated because $P_{av} \neq |\mathbf{S}_{av}|A$.

  Surface A could be the effective aperture of an antenna.
  If antenna is sensitive to only one vector component of wave, then the received power is proportional to the fraction of the total wave power contained in that component.
  If antenna has polarization completely orthogonal to polarization of wave, $P_{av} = 0$.

Boundary conditions (See Table 6-2) **– NOT COVERED ON SPRING 2024 FINAL EXAM**

- Tangential electric field intensity: $E_{1\tan} = E_{2\tan}$
- Normal electric flux density: $D_{1n} - D_{2n} = \rho_s$ (surface charge density is infinitely thin)
- Tangential magnetic field intensity: $H_{1\tan} - H_{2\tan} = J_s$ (surface current density is infinitely thin and flows in direction orthogonal to magnetic field)
- Normal magnetic flux density: $B_{1n} = B_{2n}$
- Inside a perfect conductor, the normal and tangential fields are all zero.
- Wave reflection and transmission at planar interfaces – normal incidence
    - distinction between incident, reflected, and transmitted waves
    - actual, total field on incident side of boundary is the sum of incident and reflected waves (which propagate in opposite directions)
    - at the boundary only: $\mathbf{E}^i + \mathbf{E}^r = \mathbf{E}^t$ and $\mathbf{H}^i + \mathbf{H}^r = \mathbf{H}^t$
    - definitions of reflection and transmission coefficients:

      $$\Gamma = \frac{\tilde{E}_o^r}{\tilde{E}_o^i} \quad \text{and} \quad \tau = \frac{\tilde{E}_o^t}{\tilde{E}_o^i}$$
    - expressions for Γ and τ in terms of material properties are found by applying boundary conditions
    - reflection coefficient: $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$, where material no. 1 is on the incident side of the boundary; applies to both lossless and lossy media
    - transmission coefficient: $\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$; applies to both lossless and lossy media
    - $1 + \Gamma = \tau$; Γ and τ can be complex, but $|\Gamma| < 1$, and $|\tau| < 2$
    - solution of reflection/transmission problems for normal incidence is very similar to that for transmission line problems
    - power densities of incident, reflected, and transmitted waves (when medium 1 and medium 2 are both lossless):

      $$\tilde{\mathbf{S}}_{av}^i = \hat{\mathbf{k}} \frac{|\tilde{E}_o^i|^2}{2\eta_1}, \ \tilde{\mathbf{S}}_{av}^r = -\hat{\mathbf{k}} |\Gamma|^2 \frac{|\tilde{E}_o^i|^2}{2\eta_1}, \text{ and } \tilde{\mathbf{S}}_{av}^t = \hat{\mathbf{k}} |\tau|^2 \frac{|\tilde{E}_o^i|^2}{2\eta_2}$$
    - possible for transmitted (medium 2) field to have greater magnitude than incident (medium 1) field, but power is conserved because, in that case, $|\eta_1| < |\eta_2|$
    - for lossless or lossy media, $\dfrac{1 - |\Gamma|^2}{|\eta_{c1}|} \cos\theta_{\eta 1} = \dfrac{|\tau|^2}{|\eta_{c2}|} \cos\theta_{\eta 2}$

Relevant course material:

HW:	#9
Reading:	Assignments from April 10 through April 29
Review sheets:	for Exams #1, #2, #3, and #4 (Some past material could be referred to on the final exam.)

Two supplemental sheets with Tables 2-1, 2-2, 2-4, 3-1, 3-2, and 7-1 from the textbook (Ulaby and Ravaioli, 7th ed.) plus several fundamental formulas will be made available to you during the exam. The supplement sheets are available at the ECEG 390 course Moodle site. In addition, **five** 8.5 × 11-inch two-sided handwritten help sheets may be used during the exam.

This exam will focus primarily on the course outcomes listed below and related topics:

6. Mathematically express and/or analyze the polarization of an electromagnetic wave.
7. Relate the attenuation, wavelength, and/or speed of a TEM wave propagating through a lossy medium to the medium's known constitutive parameters.

The course outcomes are listed on the Course Policies and Information sheet, which was distributed at the beginning of the semester and is available on the Syllabus and Policies page at the course web site. The outcomes are also listed on the Course Description page. Note, however, that some topics not directly related to the course outcomes could be covered on the exam as well.

Note that we did not have enough time to cover Outcome #7 (Predict the magnitudes and propagation directions of reflected and transmitted plane waves at a planar interface between two materials.)

Preparation for the Final Concept Exam

The Final Concept Exam, which constitutes 5% of the overall course grade, will assess your understanding of the major overarching ideas covered in the course. In one sense, it is difficult to suggest *preparation* strategies since the best preparation is engagement with the course material throughout the semester, and that time is now past.

However, there are two approaches that might help you solidify your understanding of the material. The first is to go over the review sheets for this exam and the prior ones carefully and make sure that you understand the basic principles encompassed in each topic. The second approach is to review the "Concepts" sections at the end of each chapter in the textbook that we covered. Remember that five supplemental readings were also assigned, so you ought to skim (or read for the first time?) that material as well. One other approach that might help is to read the introductory material at the beginning of each textbook chapter that we covered.

The Final Concept Exam will likely consist of 10–15 questions in a combination of formats such as true-false statements, multiple choice, or short written answers. A few very short (one line) calculations or interpretations of formulas might also be required. If you are well prepared, the concept exam should not take long to complete.