

## Practice Problems in Complex Arithmetic

These problems will not be collected or graded; however, I STRONGLY encourage you to work through them. I will assume in my lectures and exams that you know how to evaluate and manipulate equations involving complex math. Also, I encourage you to solve the following problems without using any complex arithmetic functions that may be built into your calculator. You should use your calculator only to perform basic arithmetic operations (addition, multiplication, squaring, etc.) and to evaluate square roots. Use the complex functions to check your answers only. Note that not all of these problems can be solved using a calculator.

In all of the following problems,  $\omega$  represents angular frequency, and  $t$  represents time.

Convert the following complex quantities from rectangular to polar form:

1.  $18 + j15$
2.  $-3 - j21$
3.  $-3 + j21$
4.  $a + jb$  ( $a$  and  $b$  are real constants)
5.  $\cos \omega t + j \sin \omega t$
6.  $\sin \omega t + j \cos \omega t$
7.  $9 \cos \omega t + j 3 \sin \omega t$

Convert the following complex quantities from polar to rectangular form:

1.  $8 e^{-j0.12}$
2.  $14 \angle 132^\circ$
3.  $-6 \angle -85^\circ$
4.  $A e^{j\omega t}$  ( $A$  is a real constant)
5.  $B e^{-j(\omega t + \pi/6)}$  ( $B$  is a real constant)
6.  $15 e^{j(\omega t - \pi/2)}$
7.  $e^{-j3\pi/2}$

The following expressions are not fully in polar form. Express them correctly in polar form:

1.  $-10 e^{j3\pi/2}$
2.  $B e^{-j\omega t - \alpha t}$  ( $B$  and  $\alpha$  are real)

Find the real and imaginary parts of the following complex quantities:

1.  $8 e^{-j0.12}$
2.  $14 \angle 132^\circ$
3.  $2$
4.  $j15$
5.  $\sin \omega t$
6.  $j \cos \omega t$
7.  $x^2 + y^2 + j 2xy$

The following expressions are not fully in rectangular form. Express them correctly in rectangular form:

1.  $j5 (-3 + j20)$
2.  $-16 (j2)^2 - j8 (j3)^2$
3.  $(j\beta + \alpha)^2$  ( $\alpha$  and  $\beta$  are real)

Find the magnitudes and phases of the following complex quantities:

1.  $e^{j\pi/2}$
2.  $5 e^{-j\pi}$
3.  $-9 e^{-j\pi/3}$
4.  $2 + j3$
5.  $-j 8$

6.  $jF e^{j0.83\pi}$  ( $F$  is a real positive constant)
7.  $jF e^{jg}$  ( $F$  and  $g$  are real positive constants)
8.  $-15$
9.  $j$

Find the complex conjugate of the following quantities:

1.  $j$
2.  $2 + j7 - Fe^{jg}$
3.  $3 \times 10^{-6}$
4.  $\text{Re}\{5 + j15\}$
5.  $\text{Im}\{5 + j15\}$
6.  $-j832 e^{-0.3z} e^{(7 - j18)t}$
7.  $8e^{-j0.12} (0.3 + j6.1)$

Some useful identities and conversion formulas:

$$e^{jx} = \cos x + j \sin x \quad e^{-jx} = \cos x - j \sin x$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{j2} \quad \cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$z = a + jb = |z|e^{j\phi} \quad \text{if} \quad |z| = \sqrt{a^2 + b^2} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\text{Also, } a = |z| \cos \phi \quad \text{and} \quad b = |z| \sin \phi$$